MONTHLY NOTICES

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ROYAL ASTRONOMICAL SOCIETY.

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Major E. H. HILLS, C.M.G., D.Sc., F.R.S., PRESIDENT, in the Chair.

The President announced that the Council had elected Miss Annie J. Cannon, of Harvard College Observatory, an Honorary Member of the Society.

R. A. Fisher, 56 Kempshott Road, Streatham, S.W.;
John Spedding Fletcher, London County Council School of Engineering and Navigation, High Street, Poplar, E.;
John Guild, A.R.C.S., D.I.C., Imperial College of Science and Technology, South Kensington, S.W.; and
Charles P. Olivier, M.A., Ph.D., Agnes Scott College, Decatur, Georgia, U.S.A.,

were balloted for and duly elected Fellows of the Society.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended:—

William Edward Curtis, A.R.C.S., D.I.C., B.Sc., Imperial College of Science, South Kensington, S.W. (proposed by A. Fowler);
Hermann Glauert, B.A., Trinity College, Cambridge (proposed by A. S. Eddington);

Major-General William du Gard Gray, C.B., 30 Alexandra Court, Queen's Gate, S.W. (proposed by A. Fowler);
Ahmed Hussain, B.A., B.L., Chief Secretary to H.H. the Nizam's Government, Hyderabad, Deccan, India (proposed by A. B. Chatwood);
Rev. Thomas Laing, United Free Church Manse, Lumsden, Aberdeenshire (proposed by Rev. Charles Whyte);
Rev. Arthur Cyril Ridsdale, M.A., Howrah, Bengal, India (proposed by H. G. Tomkins);
Charles Robinson, M.A., 64 Rydal Road, Lancaster (proposed by Rev. R. F. Rendell); and
Frank Sargent, Physics Department, Bristol University, and 16 Cottrell Road, Eastville, Bristol (proposed by H. H. Turner).

One hundred and sixteen presents were announced as having been received since the last Meeting, including, amongst others:—

Six books selected from the library of the late Sir David Gill, presented by Lady Gill.

W. W. Campbell, "Stellar motions," presented by the author; Royal Observatory, Greenwich, Photographic magnitudes of stars, presented by the Astronomer Royal.

Thirteen photographs of nebulæ and spectra, and two stereograms of Comet Morehouse (glass positives), taken at the Heidelberg Observatory, presented by Professor Max Wolf.

Astrographic Chart: twenty-two charts presented by the French Minister of Public Instruction, and thirty charts presented by the Observatory, Tacubaya, Mexico.

The Terms in the Moon's Motion depending on the Node.

By Ernest W. Brown, M.A., Sc.D., F.R.S.

1. A curious liability to error seems to have occurred in connection with computations of terms depending on the ellipticity of the Earth's figure. Corrections to certain coefficients given by Cowell and Newcomb as deductions from Hansen's theory have been made, and the published results of my own theory have also needed corrections. In the comparison of the Greenwich observations with theory, a correct transformation of Hansen's terms to ecliptic co-ordinates is needed. It seems worth while to give this in detail, so that the steps may be tested by anyone who wishes to do so, at least for the principal terms. This paper also gives the

corresponding terms derived from my theory and reduced to the value of the Earth's ellipticity which has been provisionally adopted for the purposes of the comparison.

2. Hansen's tabular values for the terms depending on the node are obtained from the following formulæ :—*

$$n\delta z = +7''\cdot760 \sin(\Omega - 4^\circ\,42') - 0''\cdot256 \cos g \sin \Omega,$$
$$s = +8''\cdot764 \sin(f + \omega + \Omega + 169^\circ\,51'),$$
$$f = g + n\delta z + 2e \sin(g + n\delta z) + \tfrac{5}{4}e^2 \sin 2(g + n\delta z),$$
$$\delta L = \delta f - \tan^2 \tfrac{1}{2} I \,.\, \delta \sin 2(f + \omega) - s \tan I \cos(f + \omega),$$
$$\delta \sin B = s + \sin I \,.\, \delta \sin(f + \omega).$$

Here g = mean anomaly, $g + \omega = F$ = principal argument of the latitude, L = longitude on the ecliptic, B = latitude, $I = 5^\circ\,8'\,39''\cdot96$. I shall only find the terms in longitude and latitude which depend on the first multiples of Ω, g, F; but in order to obtain these accurately, it is necessary to obtain at the outset certain terms depending on $2g$, 2F. Squares of δz, s may be neglected and the second term of δL, which gives rise only to terms containing 2F in their arguments, is not required.

3. For the longitude, we have

$$\delta L = \{+7''\cdot760 \sin(\Omega - 4^\circ\,42') - 0''\cdot256 \sin \Omega \cos g\}(1 + 2e \cos g + \tfrac{5}{2}e^2 \cos 2g) - 8''\cdot764 \tan I \cos(f + \omega) \sin(f + \omega + \Omega + 169^\circ\,51').$$

Express the product of the sine and cosine in the last term as the difference of two sines, and neglect that containing $2(f + \omega)$, and therefore 2F in its argument. In the other terms put $2e = \cdot110$, $\cos^2 g = \frac{1}{2} + \frac{1}{2} \cos 2g$, $\cos g \cos 2g = \frac{1}{2} \cos g + \frac{1}{2} \cos 3g$, and neglect the higher multiples of g. We then get

$$\begin{aligned}\delta L &= +7''\cdot760 \sin(\Omega - 4^\circ\,42') - 0''\cdot014 \sin \Omega + 0''\cdot394 \sin(\Omega - 10^\circ\,9') \\ &\quad + \{0''\cdot852 \sin(\Omega - 4^\circ\,42') - 0''\cdot256 \sin \Omega\} \cos g \\ &= +8''\cdot108 \sin \Omega - 0''\cdot705 \cos \Omega \\ &\quad + \{+0''\cdot593 \sin \Omega - 0''\cdot070 \cos \Omega\} \cos l,\end{aligned}$$

which are the required terms in ecliptic longitude. There is also a small term, $-0''\cdot035 \sin 2\Omega$, in $n\delta z$; the value in δL is the same. In this last expression for δL, I put $g = l$ to conform to the notation of my theory.

4. For the latitude,

$$\delta B = (1 + \tfrac{1}{2} \sin^2 B)\delta B = (1 + \gamma^2 - \gamma^2 \cos 2F)\, \delta \sin B.$$

In this I have put, with sufficient approximation, $\sin B = 2\gamma \sin F$, where $\gamma = \sin \frac{1}{2} I$.

Put for abbreviation

$$\phi = g + \omega + \Omega - 10^\circ\,9' = F + \Omega - 10^\circ\,9'.$$

* *Tables de la Lune*, pp. 1-15.

Then

$$\begin{aligned} s &= -8''\cdot764 \sin(\phi + 2e \sin g + \tfrac{5}{4} e^2 \sin 2g) \\ &= -8''\cdot764 \{(1 - e^2) \sin \phi + 2e \cos \phi \sin g\} \\ &= -8''\cdot738 \sin \phi - 0''\cdot962 \cos \phi \sin g. \end{aligned}$$

Also, recalling that $F = g + \omega$,

$$\begin{aligned} \sin I \,.\, \delta \sin(f + \omega) &= n\delta z \,.\, \sin I (\cos F - 2e \sin F \sin g) \\ &= +0''\cdot696 \cos F \sin(\Omega - 4^\circ\, 42') \\ &\quad - 0''\cdot023 \cos F \sin \Omega \cos g - 0''\cdot077 \sin F \sin(\Omega - 4^\circ\, 42') \sin g. \end{aligned}$$

Hence

$$\begin{aligned} \delta B &= (+1\cdot0020 - \cdot0020 \cos 2F) \{s + \sin I \,.\, \delta \sin(f + \omega)\} \\ &= -8''\cdot755 \sin \phi + 0''\cdot017 \cos 2F \sin \phi \\ &\qquad + 0''\cdot697 \cos F \sin(\Omega - 4^\circ\, 42') \\ &\quad - 0''\cdot964 \cos \phi \sin g - 0''\cdot023 \cos F \sin \Omega \cos g \\ &\qquad - 0''\cdot077 \sin F \sin(\Omega - 4^\circ\, 42') \sin g. \end{aligned}$$

Separate the second term into two sines and neglect that with argument $2F + \phi$, and therefore containing 3F. Replace ϕ by its value $F + \Omega - 10^\circ\, 9'$, and separate out the sines and cosines of Ω from those of F, g. We obtain—

$$\begin{aligned} \delta B = &\{-8''\cdot746 \sin(\Omega - 10^\circ\, 9') + 0''\cdot697 \sin(\Omega - 4^\circ\, 42')\} \cos F \\ &+ \{-8''\cdot764 \cos(\Omega - 10^\circ\, 9')\} \sin F \\ &+ \{+0''\cdot482 \sin(\Omega - 10^\circ\, 9') - 0''\cdot011 \sin \Omega \\ &\qquad - 0''\cdot038 \sin(\Omega - 4^\circ\, 42')\} \cos(F - g) \\ &+ 0''\cdot482 \cos(\Omega - 10^\circ\, 9') \sin(F - g) \\ &+ \{-0''\cdot482 \sin(\Omega - 10^\circ\, 9') - 0''\cdot011 \sin \Omega \\ &\qquad + 0''\cdot038 \sin(\Omega - 4^\circ\, 42')\} \cos(F + g) \\ &- 0''\cdot482 \cos(\Omega - 10^\circ\, 9') \sin(F + g) \\ = &(-7''\cdot914 \sin \Omega + 1''\cdot483 \cos \Omega) \cos F \\ &+ (-8''\cdot627 \cos \Omega - 1''\cdot544 \sin \Omega) \sin F \\ &+ (+0''\cdot425 \sin \Omega - 0''\cdot082 \cos \Omega) \cos(F - l) \\ &+ (+0''\cdot474 \cos \Omega + 0''\cdot085 \sin \Omega) \sin(F - l) \\ &+ (-0''\cdot447 \sin \Omega + 0''\cdot082 \cos \Omega) \cos(F + l) \\ &+ (-0''\cdot474 \cos \Omega - 0''\cdot085 \sin \Omega) \sin(F + l). \end{aligned}$$

This last expression is the portion in latitude, the change of notation, g to l, having been made. The principal term, that with argument $F + \Omega = L_0$, is $-8''\cdot271 \sin L_0$.

5. My formulæ for the terms in the elements depending on the argument Ω are, after the correction of the sign of the second term in $\delta\varpi$,

$$\begin{aligned} \delta L_0 &= +\ 7''\cdot317 \sin \Omega + 0''\cdot282 \sin \Omega_1, \\ \delta\varpi &= -\ 2''\cdot092 \sin \Omega - 0''\cdot840 \sin \Omega_1, \\ \delta\Omega &= +96''\cdot69 \sin \Omega + 15''\cdot58 \sin \Omega_1 + 1''\cdot86 \sin \Omega_2, \\ \delta\gamma &= -\ 4''\cdot351 \cos \Omega - 0''\cdot698 \cos \Omega_1 - 0''\cdot083 \cos \Omega_2, \end{aligned}$$

where

$$\Omega_1 = \Omega - 2^\circ\cdot3 t_c + 276^\circ\cdot2, \quad \Omega_2 = \Omega - 0\cdot9 t_c + 290^\circ\cdot1;$$

L_0 is the mean longitude, Ω is the angular distance of the node from the mean equinox, and t_c is reckoned from 1850 in centuries.

The first term only in the expression for each element depends on the Earth's ellipticity, the value 1/292·9 being used. In order to change to the adopted value 1/297·0, each of these terms must be multiplied by the factor

$$\left(\frac{1}{297} - ·001734\right) \div \left(\frac{1}{292·9} - ·001734\right) = 1 - \frac{1}{35·65}.$$

When this multiplication is performed, the four coefficients become respectively

$$+7''·112, \quad -2''·034, \quad +93''·98, \quad -4''·229.$$

For comparison with Hansen's values I reduce Ω_1, Ω_2 to the epoch 1875 by putting $t_c = ·25$, so that

$$\Omega_1 = \Omega + 275°·6, \quad \Omega_2 = \Omega + 289°·9.$$

With these substitutions, I obtain

$$\begin{aligned}
\delta L_0 &= +7''·140 \sin \Omega - 0''·281 \cos \Omega, \\
2e\delta l &= 2e(\delta L_0 - \delta\varpi) = +1''·017 \sin \Omega - 0''·122 \cos \Omega, \\
\delta F &= \delta L_0 - \delta\Omega = -89''·00 \sin \Omega + 16''·97 \cos \Omega, \\
\delta\gamma &= -4''·325 \cos \Omega - 0''·773 \sin \Omega.
\end{aligned}$$

6. The terms in longitude depending on Ω and the first multiple of l are given by $\delta L_0 + 2e\delta l \cos l$, where δL_0 and $2e\delta l$ have the values just written. To this must be added the term $-0''·038 \sin 2\Omega$ present in the true longitude (*loc. cit.*, p. 101).

7. The terms in the latitude depending on the first multiples of F, l, which are affected by the variations of the elements are

$$B = 18461''·5 \sin F - 1000'' \sin (F - l) + 1010'' \sin (F + l).$$

In forming δB it is to be remembered that each of these terms contains γ as a factor; the second and third terms also contain e, n, but the variations of these elements are insensible. Expressing the coefficients in radians, we have

$$\begin{aligned}
\delta B = {} & +·08950 \cos F \,.\, \delta F - ·00485 \cos (F - l) \,.\, \delta (F - l) \\
& + ·00490 \cos (F + l) \,.\, \delta (F + l) \\
& + \frac{\delta\gamma}{\gamma} \{·08950 \sin F - ·00485 \sin (F - l) + ·00490 \sin (F + l)\}.
\end{aligned}$$

With $\gamma = ·04489$ and the values found at the end of No. 5, we obtain

$$\begin{aligned}
\delta B = {} & (-7''·966 \sin \Omega + 1''·519 \cos \Omega) \cos F \\
& + (-8''·624 \cos \Omega - 1''·541 \sin \Omega) \sin F \\
& + (+0''·476 \sin \Omega - 0''·088 \cos \Omega) \cos (F - l) \\
& + (+0''·467 \cos \Omega + 0''·083 \sin \Omega) \sin (F - l) \\
& + (-0''·391 \sin \Omega + 0''·078 \cos \Omega) \cos (F + l) \\
& + (-0''·472 \cos \Omega - 0''·084 \sin \Omega) \sin (F + l).
\end{aligned}$$

To this must be added the terms $-0''\cdot017 \sin(F+\Omega)$ and $+0''\cdot083 \sin(F+2\Omega)$, present in the latitude (*loc. cit.*, p. 103). The principal term is that with argument $F+\Omega=L_0$; its coefficient is $\frac{1}{2}(-7''\cdot966-8''\cdot624)-0''\cdot017=-8''\cdot312$.

8. The published annual motions of the perigee and node (*loc. cit.*, p. 94) due to the ellipticity of the Earth are $+6''\cdot41$, $-6''\cdot00$, respectively. These are computed with the ellipticity $1/296\cdot3$, not as there stated with the same value as the periodic terms just considered.* When referred to the ellipticity $1/297$, they become

$$\text{perigee}, +6''\cdot38; \quad \text{node}, -5''\cdot97.$$

9. In general, if the ellipticity be denoted by $1/k$, a small change δk produces a change very nearly equal to $-2C\delta k/k$ in any coefficient C due to this cause.

Most of the computations given here have been tested by Mr. H. B. Hedrick.

Yale University:
1914 *January* 5.

The Perigee and Eccentricity of the Moon, 1750 to 1901.

By Ernest W. Brown, M.A., Sc.D., F.R.S.

1. While the constants of the Moon's motion, as determined from observation, depend on one another to a certain extent, it is convenient to separate the actual investigations as far as possible. In accordance with this idea, I have already obtained in two previous papers the mean longitude and mean latitude of the Moon.† In the former it was shown that Newcomb's value derived from occultations satisfied the Greenwich meridian observations within the limits of error, after some slight corrections. In the latter a constant correction, due to the irregularities of the limb, was found to be necessary to all latitudes. In the present paper the mean longitude of the perigee and the eccentricity are determined. In a paper to be published shortly, the inclination and the mean longitude of the node will be found, thus completing the determination of the constants of the orbit.

There are, however, other constants, mainly due to the figures of the Earth and Moon, which have to be found. While these are not always most accurately determined from the meridian

* *M.N.*, vol. lxx. p. 3.

† "The Longitude of the Moon from 1750 to 1910," *M.N.*, vol. lxxiii. pp. 692–714. "The Mean Latitudes of the Moon and Sun," *ibid.*, vol. lxxiv. pp. 156–167.

observations, the results obtained from the latter should be reconciled with those obtained from other sources. In particular, the adopted values of the compressions of the Earth and Moon require careful examination. A new source of error arises from the irregularities of the limb. The terms found are by no means insensible, and I hope to show in the succeeding paper that they can all be explained by an assumption concerning the shape of the limbs which agrees with the measured irregularities. It is for this reason that the term with the right ascension as argument has received an extended treatment here. [*Note added* 1914 *March* 5.—Since this paper was sent in, papers by J. E. de Vos van Steenwijk and Prof. E. F. van de Sande Bakhuysen dealing with this inequality have been received. Their results will be discussed in my paper on the inclination and the node.]

2. The most satisfactory result of this paper is the agreement of the two values for the eccentricity derived from the observations before and after 1850; their difference is considerably less than the probable error of the determination. The observed value of the secular acceleration of the perigee also agrees with the theoretical value within the probable error, but the difference found by Cowell and Newcomb between the observed and theoretical mean motions is confirmed, and cannot, I think, be explained by errors of observation. But here the adopted compressions of the Earth and Moon may possibly be in error sufficiently to account for the difference. The discussion will be reserved until the motion of the node and the term in latitude with the right ascension as argument have been obtained. A settlement of the question is difficult, since an error in the obliquity may sensibly affect the coefficient of this periodic term.

3. The attempt to discover the source of the periodic term $0''{\cdot}5 \cos \alpha$ led me into a much more extensive investigation of the observational errors of the Greenwich observations than I had at first intended, and has delayed the appearance of this paper by several months. While the values of the principal constants have been but little altered, the confidence in their accuracy has been much increased through the improved consistency between the results at different dates and the general agreement with those found by Newcomb from the occultations. I have been obliged to confine myself to systematic corrections of the observations in order to avoid undue delay. While these appear to be sufficient for the objects in view, more detailed corrections, particularly for personal equation, would, I think, still further improve the results.

4. As in my two previous papers, the adopted residuals are those obtained by Dr. Cowell from the Greenwich observations and corrected by him for various tabular errors. The particular set used here are those given in Table I. of his paper "The Longitude of the Moon's Perigee." * The various corrections which he has applied will be found stated there and in the references to

* *M.N.*, vol. lxv. pp. 269, 270.

his previous papers. I have completed the work in such a way that the tabular corrections shall make the adopted theory the same as that which I have published in the *Memoirs* of the Society, with some corrections noted below. The results have also been corrected for systematic errors of observation, the standard being Newcomb's Fundamental Catalogue. The residuals thus corrected are then analysed. The final section of the paper contains the concluded values of the constants and a comparison with other determinations, together with a statement concerning the empirical terms. The cost of those parts of the calculations which I have not performed myself has been met from the Lunar Tables Fund provided by Yale University.

I. *The Tabular Corrections.*

5. The perturbations of the Moon's motion are expressed in two ways: those added to the true longitude, and those added to the mean elements. One of the elements which we desire to find is the perigee, and the correction of the adopted value is obtained from the error $2e\delta l = 2e(\delta \mathrm{L} - \delta\varpi)$ of the coefficient of $\cos l$. It is therefore possible to neglect the corrections to the *mean* longitude, provided we use its uncorrected value to obtain $\delta\varpi = \delta \mathrm{L} - \delta l$. But as I wished to examine the comparison of the theoretical and observed values of the coefficient of $\cos l$ in addition to the determination of ϖ, I have added all the corrections to this coefficient and to that of $\sin l$ whatever their source, with the exception of some minute terms which would only affect means calculated to a higher degree of accuracy (Nos. 9, 10).

Besides the direct tabular errors there are those of the "allied" terms. By an examination of over 5600 observations between 1847 and 1901, Dr. Cowell has shown that the effect of a neglected term $a \sin(\alpha \pm \mathrm{D})$ is a tabular minus observed error of $-\frac{1}{2} a \sin \alpha$. In most cases the mean value of $\sin \alpha$ is zero, but there are some terms, notably those with arguments $l \pm \mathrm{D}$, as Cowell pointed out, which must be considered, as well as terms in which α has a very slow motion. I assume that the same factor of correction, $-\cdot 5$, will serve for the period from 1750 to 1850.

6. *Regular Tabular Corrections.*—For the corrections made by Dr. Cowell I refer to his papers or to No. 6 of my longitude paper. The residuals adopted here were further corrected by him.* The lists of terms which are given in the following tables and paragraphs have been compiled after a comparison of all the theories involved and the various corrections which have been made to them. Table I. contains the corrections to the Airy series.

* *M.N.*, lxv. pp. 268–275.

TABLE I.

Tabular Corrections to the Coefficients of cos l, sin l. Periods 1–89.

Argument.	Coef. of cos l.	Coef. of sin l.	
$2D-2l+2T-2J$	$-0''\cdot24 \sin$	$-0''\cdot24 \cos$	(H)
$2D-2l+2T-3J+7^\circ$	$+0''\cdot12 \sin$	$+0''\cdot12 \cos$	(H)
$2M-T+225^\circ$	$+0''\cdot15 \sin$		
$2V-3T+264^\circ$	$+0''\cdot09 \sin$		(H)
$J+282^\circ$	$+0''\cdot13 \sin$		(H)
$2D-2l+6V-8T+163^\circ$	$+0''\cdot08 \sin$	$+0''\cdot08 \cos$	(H)
$2D-2l+2l'$	$-0''\cdot09 \sin$	$-0''\cdot09 \cos$	
$2F-2l$	$+0''\cdot09 \sin$		
$\Omega+278^\circ$	$+0''\cdot124 \sin$		
$D-l+l'$	$+0''\cdot819 \sin$	$+0''\cdot515 \cos$	
$132^\circ t_c+214^\circ\cdot1$	$-1''\cdot69 \sin$		(H)
$133^\circ t_c+212^\circ\cdot4$	$+1''\cdot57 \sin$		(H)
$132^\circ t_c+93^\circ\cdot9$	$-1''\cdot70 \sin$		(H)
$131^\circ t_c+100^\circ\cdot6$	$+1''\cdot42 \sin$		(H)
$4D-3l+25M-23T+67^\circ$	$+0''\cdot03 \sin$		(H)
$l+3T-10V-2^\circ\cdot6(t^c-0\cdot5)+33^\circ$	$+0''\cdot03 \sin$		(H)
$20^\circ\cdot2(t_c-0\cdot5)+41^\circ\cdot1$	$+0''\cdot32 \sin$		(H)
$13T-8V+313^\circ\cdot9$	$+0''\cdot09 \sin$		(H)
	$+0''\cdot055$		
	$+0''\cdot212t_c$		(H)

The first eight terms call for no special mention. The term argument $\Omega+278^\circ$ is obtained in the following way. Airy had no such term in the coefficient of $\cos l$; Cowell applied $+1''\cdot000 \sin \Omega$. I find,* with the value 1/297 for the compression,

$$(+1''\cdot017 \sin \Omega - 0''\cdot123 \cos \Omega) \cos l,$$

so that the correction still required to the coefficient of $\cos l$ is

$$+0''\cdot017 \sin \Omega - 0''\cdot123 \cos \Omega = 0''\cdot124 \sin (\Omega + 278^\circ).$$

The term argument $D-l+l'$ arises from the terms with arguments $D+l'$, $D+l'-2l$ in the true longitude. My published † values for the epoch 1850 have been corrected to the epoch 1800 for the variability of e' and to the solar parallax $8''\cdot806$. Airy's coefficient of sin $(D+l')$, as corrected by Cowell, ‡ is $17''\cdot900$, with the further correction $-0''500$ §; mine is $+18''\cdot067$, giving the correction $+0''\cdot667$. That of sin $(D+l'-2l)$ is absent, but is given the value $+0''\cdot200$ by Cowell §; mine is $+0''\cdot352$, giving

* *M.N.*, vol. lxxiv., March.
† *Mem., R.A.S.*, vol. lvii. pp. 130 *et seq.*
‡ *M.N.*, vol. lxiv. p. 572.
§ *M.N.*, vol. lxv. p. 268.

the correction $+0''\cdot152$; the combination of these gives the corrections in the table.

The succeeding seven terms are those explained in No. 13 of my longitude paper, multiplied by $2e\cos l$. The last of these and the last periodic term of the table are also present in $\delta\varpi$ with coefficients $-2\cdot5$ times those in the mean longitude.* Hence for the effect of these two terms on the coefficient of $\cos l$, it is necessary to multiply the corresponding terms in the mean longitude by $3\cdot5$ and by $2e=0\cdot11$.

The correction $+0''\cdot055$ $(=+0''\cdot50\times 2e)$ is a further correction to the position of the perigee which Cowell states † is necessary to make Airy's value agree with Hansen's: this has been tested. The final correction, $+1''\cdot93t_c\times 2e$, arises from an addition to the mean longitude explained in No. 13 of my longitude paper.

The symbol t_c denotes Julian centuries from 1800.

7. The terms in Table I. followed by the letter (H) have also been applied to the Hansen series (Per. 86–133), together with the following additional corrections:—

TABLE II.

Tabular Corrections to the Coefficients of cos l, sin l. Periods 86–133.

Argument.	Coef. of cos l.	Coef. of sin l.
	"	"
$2M-T+225^\circ$	$+0\cdot11\sin$	
$2D-2l+2l'$	$-0\cdot05\sin$	$-0\cdot05\cos$
$2F-2l$	$-0\cdot25\sin$	$-0\cdot25\cos$
$\Omega+244^\circ$	$+0\cdot06\sin$	
$D-l+l'$	$+0\cdot535\sin$	$+0\cdot599\cos$

With respect to the term argument $\Omega+244^\circ$, Hansen's tabular value, reduced to the longitude on the ecliptic, is equivalent to ‡

$$(+0''\cdot593\sin\Omega-0''\cdot070\cos\Omega)\cos l.$$

The correction added by Cowell § is $+0''\cdot446\sin\Omega\cos l$. Hence the correction to the coefficient of $\cos l$ necessary to reduce to my value given in No. 6 above is

$$-0''\cdot022\sin\Omega-0''\cdot052\cos\Omega=+0''\cdot056\sin(\Omega+247^\circ).$$

The term tabulated is that given in Table II., but the difference is insensible.

For the terms, argument $D+l'-l$, my values are corrected as in No. 6 above, but to the epoch 1875. Hansen's coefficient of $\sin(D+l')$ is $+18''\cdot167$ plus Cowell's addition, $-0''\cdot700$; mine is

* I call attention again to certain errors on p. 96 of Part V. of my "Theory of the Motion of the Moon," see *M.N.*, vol. lxxiv. p. 424. In the value of δw_2, *for* $+0''\cdot840$ *read* $-0''\cdot840$, and "the ten periodic terms" should have the factor $-2\cdot5$. To δw_3 should be added the same ten terms with the factor $\cdot75$.

† *M.N.*, vol. lxv. p. 273.

‡ *M.N.*, vol. lxxiv., March.

§ *M.N.*, vol. lxiv. p. 85.

+ 18″·034, giving the correction + 0″·567. His coefficient of $\sin(D + l' - 2l)$ is + 0″·384; mine is + 0″·352, giving the correction − 0″·032. The combination of these gives the values in the table.

All these terms were tabulated, mainly by Mr. Hedrick, and are contained in the third columns of Tables III., IV.

8. *Allied Terms.*—The terms with arguments $(D + l' - l) \pm l \pm D$ will, if their coefficients are erroneous, give rise to apparent periodic errors with arguments $(D + l' - l) \pm l$, as explained in No. 5 above—that is, to corrections of the form $a \sin(D + l' - l) \cos l$, $a \cos(D + l' - l) \sin l$. My published coefficients of these were corrected for the change of e' to the epoch 1800 for the Airy series and to the epoch 1875 for the Hansen series, when this variation had not been used in the adopted tabular values. The coefficients in the Airy series (A − C) and in the Hansen series (H − C) and the corrections required to reduce them to mine are as follows:—

Argument.	A − C.	Cor. (Per. 1–89.)	H − C.	Cor. (Per. 86–133.)
l'	− 669″·630	− 0″·181	− 669″·456	− 0″·355
$2D + l'$	− 24·480	− 0·001	− 24·452	− 0·029
$l' - 2l$	− 9·600	− 0·127	− 9·719	+ 0·010
$2D + l' - 2l$	+ 2·400	+ 0·100	+ 2·556	− 0·056

The tabular corrections resulting from these are:—

to the coefficient of $\sin(D + l' - l) \cos l$,

$-\frac{1}{2}(-0''·181 - 0·001 - 0·127 + 0·100) = + 0''·105$ (Per. 1–89),
$-\frac{1}{2}(-0''·355 - 0·029 + 0·010 - 0·056) = + 0''·215$ (Per. 86–133);

and to the coefficient of $\cos(D + l' - l) \sin l$,

$-\frac{1}{2}(-0''·181 - 0·001 + 0·127 - 0·100) = + 0''·078$ (Per. 1–89),
$-\frac{1}{2}(-0''·355 - 0·029 - 0·010 + 0·056) = - 0''·169$ (Per. 86–133).

These further tabular corrections are also included in the third columns of Tables III. and IV.

Tabular Corrections not applied in Tables III., IV.

9. Certain small terms of long period which are insensible in units of 0″·1, but which slightly affect means computed to 0″·01, are more conveniently applied to the means themselves. I have examined all those in my theory which could affect the values of the perigee and eccentricity; the terms given here are alone sensible:—

In the mean longitude, the terms

$$+ 0''·108 \sin(l + 29T - 26V + 112°),$$
$$+ 0''·040 \sin\{119°(t_c - 0·5) + 152°\},$$
$$+ 0''·026 \sin(15M - 8T + 43°),$$

and in $-\delta\varpi$ the last two multiplied by 2·5. I shall apply these corrections to the sums of eight of the coefficient of cos l shown in

No. 16 below. Hence for correction of these sums the first term is to be multiplied by $8 \times 2e = +\cdot88$, and the other two by $\cdot88 \times 3\cdot5$.

In the true longitude the term $+0''\cdot050 \sin(l-\phi)$ and in $-\delta\varpi$ the term $+0''\cdot118 \sin\phi$, where $\phi = l + 16T - 18V + 151°$. These terms produce $-0''\cdot037 \sin\phi\cos l + 0''\cdot050 \cos\phi \sin l$. The former is insensible to the degree of accuracy which is possible in the determination of the motion of the perigee. The mean value of $+0''\cdot050 \cos\phi$, applicable to the tabular coefficient of $\sin l$, is

Periods 1–89, $+0''\cdot033$; Periods 86–133, $-0''\cdot032$.

10. The argument of the term $+0''\cdot176 \sin(4T - 3V + 272°)$ in the true longitude may be written $(-l + D) + (5T - 3V - \varpi + 92°)$. The period of the second portion is 96 years. It is therefore an "allied" term. The tabular correction to the coefficient of $\cos l$ is $-0''\cdot088 \sin(5T - 3V - \varpi + 92°)$, which will be applied with those terms mentioned at the beginning of this section. The mean value of the tabular – observed correction to the coefficient of $\sin l$ is

Periods 1–89, $-0''\cdot001$; Periods 86–133, $-0''\cdot045$.

The term $0''\cdot048 \sin(4M - 3T + 212°)$, similarly treated, gives a zero correction to the coefficient of $\cos l$, and to the tabular – observed value of the coefficient of $\sin l$, the mean correction

Periods 1–89, $+0''\cdot005$; Periods 86–133, $-0''\cdot009$.

My coefficients of the allied terms $\sin(l \pm D)$ are $+18''\cdot609$, $-8''\cdot466$, respectively, after correction to the adopted value of the solar parallax. The adopted coefficients in the Airy series are $+18''\cdot600$, $-8''\cdot200$, requiring corrections $+0''\cdot009$, $-0''\cdot266$. Hansen's coefficients are $+19''\cdot151$, $-8''\,543$, to the former of which Cowell applied $-0''\cdot550$; the corrections are, therefore, $+0''\cdot008$, $+0''\cdot077$. Hence the correction to the tabular – observed coefficients on account of these two allied terms are

Periods 1–89, $+0''\cdot128$; Periods 86–133, $-0''\cdot043$.

II. *Observational Corrections.*

11. *The Parallax of the Moon.*—I have discussed this in my paper on the mean latitudes of the Sun and Moon.* The theoretical value of the constant of sine parallax at Greenwich was found to be $3415''\cdot44$. Airy's adopted value for the same constant is †

$$3421''\cdot80 \times [9\cdot9991136] = 3414''\cdot82,$$

so that the correction required is $+0''\cdot62$. The correction to the right ascension α is zero, since all observations are made on the meridian; that to the declination is $+0''\cdot62(\sin\phi - \cdot4\cos\phi\sin\alpha)$, where ϕ is the latitude of Greenwich. This gives $+0''\cdot48 - 0''\cdot15 \sin\alpha$. To get the correction to the longitude, we apply the factor

* *M.N.*, vol. lxxiv. p. 160.

† *Reduction of Greenwich Observations of the Moon*, vol. i. p. xxii.

·4 cos a. The term with argument $2a$ may be neglected; the correction is then $+0''\cdot19\cos a$ to the observed longitudes. This has been applied.

The corrections on account of parallax to the observed declinations found in No. 9 of my latitude paper were $+0''\cdot07$ from 1847–1861 and $+0''\cdot28$ from 1862–1901. The corrections applied during these intervals to the observed longitudes were therefore $+0''\cdot03\cos a$ and $+0''\cdot11\cos a$ respectively.

12. *Corrections to the Declination System.*—Any error in the declination system affects the longitude through the factor ·4 cos a, by which small errors of declination are multiplied when we transform from equatorial to ecliptic co-ordinates. It is not clear what *general* correction should be applied to the observations from 1750 to 1830. It was obviously impossible to correct the individual observations without undertaking an investigation of a length out of all proportion to the importance of the object in view. But as Airy made an effort to reduce all observations to the places given in the *Tabulæ Regiomontanæ*, an approximate value may be obtained by finding the correction necessary to reduce these tables to Newcomb's Fundamental Catalogue. Using the comparison given by Safford,* I find that the correction necessary to reduce these declinations to Boss's system is approximately

$$d\delta = +1''\cdot00 + 0''\cdot37t_c,$$

where $t_c = 0$ at 1800. The reduction of Boss's system to that of Newcomb is given by the latter on p. 213 of his Fundamental Catalogue; I deduced the value $d\delta = -0''\cdot10 + 0''\cdot20t_c$ for this. The combination with the former correction gives

$$d\delta = +0''\cdot90 + 0''\cdot57t_c.$$

This, multiplied by $-\cdot4\cos a$, has been applied to the tabular – observed mean residuals of each period of 414 days from 1750 to 1825.

From 1825 on, Airy does not appear to have compared the Moon with star places, but to have used the Greenwich determinations of zenith point and colatitude. An approximate correction will therefore be obtained from the comparison† of the Greenwich Catalogue with Newcomb's Fundamental Catalogue, using the former for the years covered by the observations. I finally adopted for 1825–35, $+1''\cdot50$; 1836–41, $+0''\cdot50$; 1842–50, $+0''\cdot24$, as the changes required for the observed declinations; but I am not at all clear that the evidence for these corrections is very good.

From 1851 to 1901 I used the corrections given in No. 7 of my mean latitude paper. These are corrections to north polar distances and require a change of sign for application to the observed declinations.

After all these changes had been multiplied by ·4 cos a, they

* *Amer. Eph. Papers*, vol. ii. p. 85.
† *Ibid.*, vol. viii. pp. 232, 233.

were added to the corrections for parallax, and were tabulated in the form $a \cos \varpi \cos l - a \sin \varpi \sin l$. The results are given in the fourth columns of Tables III., IV.

13. *Right Ascension Correction.*—It is well known that many catalogues of stars require corrections depending on the right ascensions; these are usually analysed into Fourier series having the right ascension as argument. We are only concerned here with the term having the first multiple of this angle, since this term alone will affect the means of the coefficients of sin l, cos l over 414 days; the period will be that of the perigee. Since these catalogues are used for time determinations, the right ascension of the Moon will be similarly affected during the periods in which the catalogues were in use.

Airy compared the right ascensions of the Moon from 1750 to 1830 with the stars of the *Tabulæ Regiomontanæ*, and they therefore carry the same errors as that catalogue. Newcomb has given the comparisons for each star with his Fundamental Catalogue * in the form $a + bt_c + ct_c^2 + dt_c^3$. Mr. Hedrick analysed these for four dates by means of the formulæ developed by Newcomb.† He found the following corrections:—

	s	s	s
1750	$-0^s{\cdot}008$	$-0^s{\cdot}013 \cos a$	$+0^s{\cdot}019 \sin a$,
1800	$+0^s{\cdot}016$	$+0^s{\cdot}007 \cos a$	$+0^s{\cdot}003 \sin a$,
1850	$+0^s{\cdot}037$	$+0^s{\cdot}021 \cos a$	$-0^s{\cdot}016 \sin a$,
1900	$+0^s{\cdot}053$	$+0^s{\cdot}041 \cos a$	$-0^s{\cdot}032 \sin a$.

The run of the differences is quite satisfactory. The constant terms have already been used in the investigation of the mean longitude. I find that the periodic parts can be well represented by

$$+ (0^s{\cdot}005 + 0^s{\cdot}034 t_c) \cos a + (0^s{\cdot}002 - 0^s{\cdot}035 t_c) \sin a,$$

giving a correction

$$+ (0''{\cdot}08 + 0''{\cdot}51 t_c) \cos a + (0''{\cdot}03 - 0''{\cdot}52 t_c) \sin a$$

to the observed places; t_c is zero at 1800. This has therefore been applied to the residuals from 1750 to 1830.

Airy did not re-reduce the observations from 1831 to 1851, but took the published results and applied only constant corrections to the equinoctial points, the star places for time being taken from the *Nautical Almanac*. During the period 1831 to 1847 Pond's Catalogue of 1830 was used with corrections from 1843 to 1847 based on the observations during the years 1836 to 1839. It seemed to be an unnecessary refinement to analyse these corrections for so short a period. The analysis of the errors of Pond's Catalogue given by Newcomb on p. 232 of his Fundamental Catalogue produces a periodic correction to the observed places of

$$+ 0''{\cdot}27 \sin (a + 185^\circ).$$

* *Washington Observations*, 1870, App. III. p. 36.
† *Ibid.*, p. 40.

From 1848 to 1901 the errors are those of the various Greenwich Catalogues for the years they were in use. The analysis of these errors depending on the right ascension (*loc. cit.*, p. 160) gave

Cat.	Per. in use.	Cor. to Obs. R.A.	
1840	1848	$+0''.03 \cos \alpha$	$-0''.59 \sin \alpha$
1845	1849–55	+0·24	−0·29
1850	1856–61	+0·16	−0·20
1860	1862–69	+0·31	−0·21
1864	1870–77	+0·15	−0·14
1872	1878–88	+0·12	−0·05
1880	1889–92	+0·05	−0·06
1890	1893–1901	+0·05	0·00

All these corrections were computed in the form

$$a \cos(\alpha + \phi) = a \cos(\varpi + \phi) \cos l - a \sin(\varpi + \phi) \sin l,$$

interpolated where necessary for the various periods P, and are placed in the fifth columns of Tables III., IV.

14. *The Empirical Terms in the Mean Longitude.*—The coefficient of $\cos l$ is $2e\delta l = 2e(\delta L - \delta\varpi)$. In order to find $2e\delta\varpi$, it is therefore necessary to subtract whatever expression may be found for $2e\delta l$ from $2e\delta L$. The great empirical term in L has already been inserted in the coefficient of $\cos l$. But the mean longitude still contains empirical terms of shorter period, and the question arises as to whether $\delta\varpi$ contains such terms. An early analysis of the coefficients of $\cos l$ gave a term which agreed well with the 60- to 70-year term in mean longitude multiplied by $2e$. I conclude, therefore, that this term is not present in ϖ.

It is not possible to separate an empirical term from observational errors, but the comparison with Newcomb's occultation results showed that the latter are, in the mean, small compared with the former, although at certain dates this is probably not the case. If, then, we subtract $2e\delta L$ from the coefficients of $\cos l$, where δL represents the residuals of the mean longitude as shown in the columns "Mer." of Tables II., III. in my longitude paper, we eliminate the empirical terms from the coefficient of $\cos l$, but probably introduce occasional large errors of observation, especially between 1750 and 1840. These errors are almost certainly sometimes as large as $2''$ in a period of 414 days, producing errors of $0''.2$ in the coefficient of $\cos l$. They probably affect the determination of the motion of the perigee very little, but will make the analysed coefficients of $\cos \alpha$ and $\sin \alpha$ (Nos. 18, 19 below) differ at certain dates very considerably from their mean values. Nevertheless it seemed better to allow the defect than to make a mean correction and so introduce possible systematic errors of a periodic nature.

The correction is tabulated in the sixth columns of Tables III.

III. *The Final Residuals.*

15. Tables III. (*a*), (*b*) and IV. (*a*), (*b*) contain the results of the foregoing investigations into the coefficients of $\cos l$, $\sin l$, each separated into two parts corresponding to the Airy series (periods 1–89) and Hansen series (periods 86–133 respectively). In each table the first column contains the number of the period; the second column, the residuals given by Cowell (No. 6); the third column, the tabular corrections (Nos. 6–8): the fourth column, the parallax and declination corrections (Nos. 11–12); the fifth column, the right ascension correction; and the sixth columns of Tables III., the value of $-2e\delta L$ (No. 14). The next column (Br.) contains the sum of the preceding columns beginning with the second, and therefore the residuals to be analysed. The final column shows the residuals after the subtraction from the previous column of the results of the analysis which follows. In the case of $\cos l$ the subtracted portion is

$$+0''\cdot03+0''\cdot12t_c-0''\cdot57t_c^2+0''\cdot50\cos\varpi,$$

and for $\sin l$ it is $-0''\cdot50\sin\varpi$.

The tabular corrections were actually computed in units of a thirtieth of a second of arc, and the observed corrections in units of $0''\cdot01$.

TABLE III. (*a*)

Coefficients of cos l. Units of $0''\cdot1$. *Tabular – Observed Residuals of the Airy Series.*

Per.	C.	Tab.	π, δ.	R.A.	$-2e\delta L$.	Br.	Res.
1	+ 4	+ 8	− 3	− 1	− 3	+ 5	0
2	− 3	+ 1	0	− 3	+ 1	− 4	− 5
3	+ 2	− 6	+ 3	− 3	+ 1	− 3	0
4	− 3	−10	+ 4	− 2	0	−11	− 7
5	+ 6	− 6	+ 3	+ 1	+ 1	+ 5	+ 7
6	+ 3	− 1	0	+ 3	0	+ 5	+ 4
7	− 1	+ 7	− 3	+ 3	− 1	+ 5	0
8	− 1	+10	− 4	+ 1	− 2	+ 4	− 2
9	+ 6	+ 7	− 3	− 1	− 1	+ 8	+ 4
10	+12	+ 3	+ 1	− 3	− 2	+11	+11
11	+ 1	− 4	+ 4	− 2	− 1	− 2	+ 1
12	−13	− 6	+ 5	− 1	− 2	−17	−13
13	−14	− 2	+ 3	+ 1	+ 1	−11	− 9
14	−12	+ 4	− 1	+ 2	− 1	− 8	−10
15	− 6	+10	− 4	+ 2	− 1	+ 1	− 4
16	+ 4	+11	− 4	0	0	+11	+ 5
17	0	+ 7	− 2	− 1	+ 1	+ 5	+ 2
18	−10	− 1	+ 1	− 2	+ 2	−10	− 9
19	+ 2	−10	+ 4	− 1	0	− 5	0

TABLE III. (*a*)—*continued.*

Per.	C.	Tab.	π, δ.	R.A.	$-2e\delta$L.	Br.	Res.
20	+ 7	−10	+ 5	0	− 1	+ 1	+ 6
21	+ 6	− 3	+ 2	+ 1	0	+ 6	+ 8
22	− 6	+ 3	− 2	+ 2	+ 1	− 2	− 4
23	− 9	+10	− 4	+ 1	+ 2	0	− 5
24	− 6	+14	− 4	0	0	+ 4	− 1
25	− 6	+11	− 2	− 1	0	+ 2	0
26	+ 8	+ 3	+ 2	− 1	+ 3	+15	+17
27	+26	− 5	+ 5	− 1	+ 1	+26	+31
28	− 3	− 5	+ 4	0	0	− 4	0
29	+ 4	− 2	+ 1	+ 1	+ 2	+ 6	+ 8
30	− 3	0	− 3	+ 1	+ 2	− 3	− 5
31	+ 1	+ 6	− 5	0	+ 1	+ 3	− 2
32	−17	+ 7	− 4	− 1	+ 1	−14	−18
33	+ 6	+ 1	− 1	− 1	+ 1	+ 6	+ 5
34	−16	− 7	+ 3	− 1	+ 3	−18	−15
35	− 9	−10	+ 5	0	+ 3	−11	− 6
36	− 1	− 6	+ 4	+ 1	+ 3	+ 1	+ 5
37	− 7	− 1	+ 1	+ 1	+ 3	− 3	− 3
38	+ 4	+ 5	− 3	0	+ 3	+ 9	+ 6
39	+ 1	+11	− 5	0	+ 3	+10	+ 5
40	0	+ 9	− 4	− 1	+ 2	+ 6	+ 2
41	−12	+ 2	0	− 1	− 2	−13	−13
42	− 1	− 3	+ 4	0	0	0	+ 3
43	2	− 9	+ 5	+ 1	+ 1	− 4	+ 1
44	−11	− 5	+ 4	+ 1	0	−11	− 8
45	− 9	+ 2	0	0	0	− 7	− 7
46	− 5	+ 7	− 4	− 1	− 1	− 4	− 8
47	+15	+11	− 5	− 1	− 1	+19	+14
48	+ 2	+ 8	− 3	− 1	− 2	+ 4	+ 1
49	−10	+ 2	+ 1	0	− 2	− 9	− 8
50	− 8	− 5	+ 4	+ 1	− 1	− 9	− 5
51	−14	−10	+ 6	+ 1	− 1	−18	−13
52	−15	− 3	+ 3	+ 1	0	−14	−11
53	+ 1	+ 4	− 1	0	− 1	+ 3	+ 2
54	− 6	+10	− 5	− 1	− 2	− 4	− 8
55	− 5	+15	− 5	− 1	− 3	+ 1	− 4
56	− 7	+13	− 3	0	− 3	0	− 3
57	− 8	+ 5	+ 1	+ 1	− 4	− 5	− 4
58	+ 7	− 3	+ 5	+ 1	− 3	+ 7	+11
59	+ 3	− 6	+ 6	+ 1	− 3	+ 1	+ 6
60	− 3	− 2	+ 3	0	− 3	− 5	− 3

TABLE III. (*a*)—*continued.*

Per.	C.	Tab.	π, δ.	R.A.	$-2e\delta$L.	Br.	Res.
61	+13	+ 1	− 2	− 1	− 4	+ 7	+ 5
62	− 4	+ 5	− 5	− 2	− 4	− 10	− 14
63	0	+ 8	− 5	− 1	− 3	− 1	− 6
64	+ 6	+ 4	− 2	0	− 3	+ 5	+ 3
65	− 2	− 4	+ 2	+ 2	− 2	− 4	− 2
66	+ 4	− 8	+ 8	+ 2	− 1	+ 5	+ 10
67	− 1	− 7	+ 6	+ 1	− 2	− 3	+ 1
68	+ 1	− 1	+ 3	− 1	− 1	+ 1	+ 3
69	+ 1	+ 4	− 4	− 2	− 2	− 3	− 5
70	− 3	+ 9	− 8	− 2	− 1	− 5	− 10
71	+11	+10	− 6	− 1	− 2	+12	+ 8
72	+ 3	+ 3	− 1	+ 3	− 2	+ 6	+ 5
73	−10	− 4	+ 4	+ 2	− 3	−11	− 8
74	+ 1	− 8	+ 8	0	− 2	− 1	+ 4
75	+ 1	− 6	+ 5	− 2	− 2	− 4	0
76	+ 4	0	+ 1	− 3	− 3	− 1	0
77	+11	+ 6	− 2	− 2	− 3	+10	+ 7
78	0	+11	− 4	0	− 3	+ 4	− 1
79	+ 2	+11	− 2	+ 2	− 2	+11	+ 7
80	− 9	+ 4	0	+ 3	− 2	− 4	− 4
81	+ 7	− 3	+ 2	+ 2	− 1	+ 7	+10
82	+ 9	− 8	+ 3	0	− 1	+ 3	+ 8
83	− 5	− 8	+ 2	− 2	0	−13	−10
84	0	+ 2	0	− 3	+ 1	0	0
85	+ 3	+ 7	− 2	− 2	0	+ 6	+ 2
86	− 9	+14	− 3	+ 1	0	+ 3	− 2
87	−11	+15	− 2	+ 3	+ 1	+ 6	+ 3
88	−15	+ 9	0	+ 3	+ 1	− 2	− 1
89	−13	+ 3	+ 2	+ 4	0	− 4	0

TABLE III. (*b*).

The Same for the Hansen Series.

Per.	C.	Tab.	π, δ.	R.A.	$-2e\delta$L.	Br.	Res.
86	−10	+13	− 1	+ 1	+ 1	+ 4	− 1
87	− 5	+10	− 1	+ 3	+ 2	+ 9	+ 6
88	−18	+10	0	+ 3	+ 1	− 4	− 3
89	−15	+ 1	+ 1	+ 4	+ 2	− 7	− 3
90	− 5	− 2	+ 2	+ 2	+ 3	0	+ 4
91	− 9	+ 2	+ 1	− 1	+ 3	− 4	− 2
92	− 1	+ 3	0	− 3	+ 3	+ 2	0
93	0	+ 9	− 2	− 3	+ 3	+ 7	+ 2

TABLE III. (*b*)—*continued.*

Per.	C.	Tab.	π, δ.	R.A.	$-2e\delta$L.	Br.	Res.
94	− 3	+10	− 2	− 1	+ 3	+ 7	+ 1
95	− 4	+ 7	− 1	0	+ 3	+ 5	+ 1
96	−12	+ 3	+ 1	+ 2	+ 3	− 3	− 3
97	− 5	− 7	+ 2	+ 2	+ 3	− 5	− 2
98	− 6	− 6	+ 1	+ 1	+ 3	− 7	− 3
99	− 5	+ 1	+ 1	− 1	+ 4	0	+ 1
100	+ 5	+ 1	− 1	− 3	+ 4	+ 6	+ 3
101	− 3	+ 9	− 3	− 4	+ 4	+ 3	− 3
102	− 6	+11	− 2	− 2	+ 3	+ 4	− 2
103	− 5	+ 5	− 1	+ 1	+ 2	+ 2	− 1
104	− 5	+ 2	0	+ 3	+ 2	+ 2	+ 3
105	− 4	− 4	− 1	+ 4	+ 1	− 4	0
106	0	− 2	+ 1	+ 1	+ 1	+ 1	+ 4
107	+ 3	+ 2	0	− 1	0	+ 4	+ 4
108	+ 1	+ 4	0	− 2	0	+ 3	− 1
109	− 7	+13	+ 1	− 2	− 1	+ 4	− 3
110	− 3	+ 9	+ 1	− 1	− 1	+ 5	− 1
111	+ 5	+ 3	0	+ 1	− 2	+ 7	+ 4
112	+ 2	− 1	− 1	+ 2	− 3	− 1	0
113	+ 5	− 9	+ 1	+ 1	− 3	− 5	− 2
114	+ 8	− 7	+ 2	+ 1	− 2	+ 2	+ 4
115	+ 8	− 2	0	0	− 3	+ 3	+ 2
116	+ 7	+ 4	− 2	− 1	− 2	+ 6	+ 1
117	0	+13	− 3	− 1	− 3	+ 6	− 1
118	+ 5	+13	− 2	− 1	− 2	+13	+ 7
119	− 9	+11	0	0	− 3	− 1	− 3
120	−11	+ 8	+ 3	+ 1	− 3	− 2	− 1
121	0	+ 1	+ 3	+ 1	− 3	+ 2	+ 4
122	− 1	+ 3	+ 2	+ 1	− 3	+ 2	+ 2
123	− 3	+ 7	0	− 1	− 3	0	− 3
124	0	+ 7	− 2	− 1	− 3	+ 1	− 6
125	+ 6	+13	− 2	0	− 3	+14	+ 6
126	+ 4	+ 7	− 2	0	− 4	+ 5	− 1
127	+ 4	+ 3	0	0	− 4	+ 3	0
128	+ 1	− 1	+ 2	0	− 3	− 1	0
129	+ 1	− 7	+ 2	0	− 3	− 7	− 5
130	+ 4	0	+ 1	0	− 3	+ 2	+ 2
131	+ 1	+ 5	0	0	− 2	+ 4	0
132	+ 3	+ 9	− 1	0	− 2	+ 9	+ 1
133	− 7	+16	− 2	0	− 1	+ 6	− 3

TABLE IV. (*a*).

Coefficients of sin l. Units of 0″·1. *Tabular – Observed Residuals of the Airy Series.*

Per.	C.	Tab.	π, δ.	R.A.	Br.	Res.
1	− 2	− 4	+ 3	− 3	− 6	− 2
2	−10	− 8	+ 4	− 2	−16	−11
3	−10	− 7	+ 3	+ 1	−13	−10
4	+ 4	− 1	0	+ 3	+ 6	+ 6
5	+ 4	+ 4	− 3	+ 3	+ 8	+ 4
6	+ 6	+ 6	− 4	+ 1	+ 9	+ 4
7	+ 4	+ 6	− 3	− 1	+ 6	+ 3
8	− 4	+ 3	0	− 3	− 4	− 3
9	− 4	− 2	+ 4	− 3	− 5	− 1
10	− 1	− 5	+ 5	− 1	− 2	+ 3
11	−17	− 3	+ 3	+ 1	−16	−13
12	+ 7	+ 1	− 1	+ 2	+ 9	+ 8
13	− 7	+ 3	− 4	+ 2	− 6	−10
14	+ 1	+ 3	− 5	0	− 1	− 6
15	+ 1	+ 1	− 2	− 1	− 1	− 3
16	+ 7	− 3	+ 1	− 2	+ 3	+ 5
17	− 5	− 7	+ 4	− 2	−10	− 5
18	− 1	− 9	+ 5	0	− 5	0
19	− 5	− 4	+ 2	+ 1	− 6	− 4
20	+ 5	+ 1	− 1	+ 2	+ 7	+ 5
21	− 2	+ 6	− 4	+ 1	+ 1	− 4
22	+ 2	+ 7	− 5	0	+ 4	0
23	− 1	+ 6	− 2	− 1	+ 2	0
24	+ 6	+ 2	+ 2	− 2	+ 8	+10
25	+ 9	− 2	+ 5	− 1	+11	+16
26	+14	3	+ 5	0	+16	+20
27	− 6	0	+ 2	+ 1	− 3	− 2
28	− 1	+ 4	2	+ 1	+ 2	− 1
29	− 2	+ 5	− 5	0	− 2	− 7
30	+ 3	+ 4	− 4	− 1	+ 2	− 2
31	− 1	+ 1	− 1	− 1	− 2	− 3
32	− 2	− 3	+ 3	− 1	− 3	0
33	− 2	− 7	+ 5	0	− 4	+ 1
34	+17	− 7	+ 4	+ 1	+15	+19
35	− 2	− 1	+ 1	+ 1	− 1	− 1
36	− 4	+ 3	− 3	+ 1	− 3	− 6
37	− 9	+ 6	− 5	0	− 8	−13
38	+ 4	+ 6	− 4	− 1	+ 5	+ 1

TABLE IV. (*a*)—*continued.*

Per.	C.	Tab.	π, δ.	R.A.	Br.	Res.
39	+ 9	+ 4	0	− 1	+ 12	+ 12
40	− 5	− 1	+ 3	0	− 3	+ 1
41	− 11	− 6	+ 5	+ 1	− 11	− 6
42	− 7	− 5	+ 4	+ 1	− 7	− 4
43	+ 4	− 1	0	0	+ 3	+ 3
44	− 2	+ 1	− 4	0	− 5	− 9
45	+ 8	+ 2	− 5	− 1	+ 4	− 1
46	+ 12	+ 1	− 4	− 1	+ 8	+ 5
47	+ 11	− 2	0	0	+ 9	+ 10
48	− 3	− 7	+ 4	+ 1	− 5	− 1
49	0	− 10	+ 6	+ 1	3	+ 2
50	0	− 6	+ 3	+ 1	− 2	+ 1
51	0	0	1	0	1	− 2
52	+ 6	+ 5	− 5	− 1	+ 5	+ 1
53	+ 9	+ 7	− 5	− 1	+ 10	+ 5
54	− 5	+ 7	− 3	− 1	− 2	− 4
55	− 6	+ 3	+ 1	+ 1	− 1	+ 1
56	+ 6	− 1	+ 5	+ 1	+ 11	+ 15
57	− 5	− 4	+ 6	+ 1	− 2	+ 3
58	+ 4	− 1	+ 3	0	+ 6	+ 8
59	− 2	+ 3	− 2	− 1	− 2	− 4
60	+ 2	+ 5	− 5	− 2	0	− 5
61	− 4	+ 5	− 5	− 1	− 5	− 10
62	− 7	+ 3	− 2	0	− 6	− 8
63	− 2	− 1	+ 2	+ 2	+ 1	+ 3
64	0	− 6	+ 5	+ 2	+ 1	+ 6
65	− 3	− 6	+ 6	+ 1	− 2	+ 2
66	+ 2	− 1	+ 2	0	+ 3	+ 5
67	− 9	+ 4	− 4	− 2	− 11	− 14
68	− 1	+ 7	− 8	− 2	− 4	− 9
69	− 2	+ 7	− 6	− 1	− 2	− 6
70	− 2	+ 5	− 3	+ 1	+ 1	0
71	+ 1	0	+ 4	+ 2	+ 7	+ 10
72	− 2	− 4	+ 8	0	+ 2	+ 7
73	− 9	− 5	+ 6	− 2	− 10	− 6
74	+ 7	− 3	+ 1	− 3	+ 2	+ 3
75	+ 12	0	− 4	− 2	+ 6	+ 3
76	+ 19	+ 1	− 5	0	+ 15	+ 10
77	+ 4	+ 1	− 3	+ 2	+ 4	0
78	0	− 2	0	+ 3	+ 1	+ 1

TABLE IV. (*a*)—*continued.*

Per.	C.	Tab.	π, δ.	R.A.	Br.	Res.
79	+ 2	− 6	+ 3	+ 2	+ 1	+ 5
80	0	− 9	+ 4	0	− 5	0
81	+ 2	− 8	+ 2	− 2	− 6	− 3
82	− 3	− 2	0	− 3	− 8	− 8
83	− 6	+ 3	− 2	− 2	− 7	− 11
84	− 1	+ 6	− 3	0	+ 2	− 3
85	− 6	+ 6	− 2	+ 2	0	− 3
86	− 4	+ 3	0	+ 3	+ 2	+ 3
87	− 6	− 2	+ 2	+ 4	− 2	+ 2
88	− 6	− 6	+ 3	+ 2	− 7	− 2
89	+ 5	− 4	+ 2	− 1	+ 2	+ 5

TABLE IV. (*b*).

The Same for the Hansen Series.

Per.	C.	Tab.	π, δ.	R.A.	Br.	Res.
86	− 3	+ 1	0	+ 3	+ 1	+ 2
87	− 5	− 2	+ 1	+ 4	− 2	+ 2
88	0	− 6	+ 1	+ 2	− 3	+ 2
89	+ 4	− 7	+ 1	− 1	− 3	0
90	+ 5	+ 2	0	− 3	+ 4	+ 3
91	+ 1	+ 4	− 2	− 4	− 1	− 5
92	+ 2	+ 6	− 2	− 2	+ 4	− 1
93	− 7	+ 6	− 1	+ 1	− 1	− 4
94	− 1	− 3	0	+ 2	− 2	0
95	− 5	− 5	+ 2	+ 2	− 6	− 2
96	− 5	− 7	+ 3	+ 1	− 8	− 3
97	0	− 4	+ 1	− 1	− 4	− 2
98	+ 3	+ 6	− 1	− 2	+ 6	+ 4
99	+ 1	+ 8	− 2	− 4	+ 3	− 2
100	− 3	+ 11	− 3	− 2	+ 3	− 2
101	− 4	+ 10	− 1	+ 1	+ 6	+ 4
102	− 3	0	+ 1	+ 3	+ 1	+ 3
103	− 2	− 2	+ 1	+ 4	+ 1	+ 6
104	+ 2	− 7	0	+ 2	− 3	+ 1
105	+ 3	− 5	0	− 1	− 3	− 1
106	0	+ 3	+ 1	− 2	+ 2	0
107	0	+ 3	+ 1	− 2	+ 2	− 3
108	+ 2	+ 4	+ 1	− 1	+ 6	+ 2
109	+ 1	+ 2	0	+ 1	+ 4	+ 3

TABLE IV. (*b*)—*continued.*

Per.	C.	Tab.	π, δ.	R.A.	Br.	Res.
110	− 1	− 7	− 1	+ 2	− 7	− 4
111	+ 2	− 9	− 1	+ 2	− 6	− 1
112	+ 6	−10	− 1	+ 1	− 4	0
113	+ 1	− 5	0	0	− 4	− 3
114	+ 4	+ 5	− 1	− 1	+ 7	+ 4
115	+ 5	+ 6	− 2	− 1	+ 8	+ 3
116	− 7	+ 8	− 3	− 1	− 3	− 7
117	− 3	+ 5	0	0	+ 2	+ 2
118	+ 1	− 5	+ 2	+ 1	− 1	+ 2
119	− 5	− 6	+ 3	+ 1	− 7	− 2
120	+ 2	− 8	+ 2	+ 1	− 3	0
121	+ 6	− 4	0	0	+ 2	+ 2
122	+ 4	+ 5	− 2	− 1	+ 6	+ 2
123	− 1	+ 4	− 3	0	0	− 5
124	+ 3	+ 5	− 2	0	+ 6	+ 3
125	− 3	0	0	+ 1	− 2	− 1
126	− 4	− 8	+ 2	+ 1	− 9	− 5
127	− 1	− 6	+ 2	+ 1	− 4	+ 1
128	− 3	− 6	+ 2	0	− 7	− 4
129	+ 3	+ 2	0	0	+ 5	+ 4
130	− 5	+10	− 1	0	+ 4	0
131	− 3	+ 9	− 1	0	+ 5	0
132	− 2	+10	− 1	0	+ 7	+ 4
133	− 6	+ 4	0	0	− 2	0

IV. *Analysis of the Residuals.*

16. *The Coefficients of cos l.*—In order to obtain the secular parts it is better to eliminate, as far as possible, any periodic terms which may be present. The residuals to be treated are those in the columns headed Br. in Tables III. (*a*), (*b*). The graph of these shows, at any rate in the later portions, a well-marked term of period about 8P, or nine years, and a coefficient about $0''{\cdot}5$. To eliminate this, I took sums of 8 consecutive residuals, beginning at P = 6 in order to get an exact number of sums of 8 out of the 133 residuals. The sums of the overlapping portions, P = 86–89, only differ by one unit of $0''{\cdot}1$.

These have to be further corrected for the small terms of very long period which have not been inserted (Nos. 9, 10). The following table shows the sums of 8, the corrections, the residuals calculated from the formula obtained below, and the outstanding residuals after the removal of the calculated secular parts.

TABLE V.

Sums of 8 *of Coefficients of cos l. Units of* 0″·1.

Mean P.	ΣBr.	Cor.	Cal.	Rem.
9°5	+ 3	+ 1	+ 8	− 4
17°5	+ 1	+ 1	+ 4	− 2
25°5	+47	+ 2	+ 1	+48
33°5	−39	+ 3	− 1	−35
41°5	−10	+ 3	− 2	− 5
49°5	−28	+ 3	− 3	−22
57°5	+ 2	+ 1	− 3	+ 6
65°5	−10	+ 1	− 2	− 7
73°5	+ 6	0	0	+ 6
81°5	+14	− 1	+ 2	+11
89°5	+ 7	− 1	+ 4	+ 2
97°5	+ 6	− 1	+ 8	− 3
105°5	+16	− 1	+11	+ 4
113°5	+23	+ 1	+16	+ 8
121°5	+29	+ 1	+21	+ 9
129°5	+21	0	+27	− 6

The least square analysis of the sum of the columns Σ Br. + Cor. gives, for the correction of the adopted value of l,

$$-\delta l = -0''\cdot28 - 1''\cdot1 t_c + 5''\cdot2 t_c^2,$$

where t_c is the number of centuries from 1800. In finding this result, the observations before 1847 (period 86) were given half-weight. The probable error of this result can best be judged from the last column of Table V.; the numbers in that column can be expressed as errors of δl if we multiply them by $1'' \div 8\cdot8$.

17. *The Coefficients of sin l.*—As with the coefficients of cos l, I take sums of 8 to eliminate the term of period 8P. The eleven sums of 8 for the Airy series, periods 1–88, are

$$-10,\ -19,\ +1,\ +21,\ +13,\ -4,\ +17,\ -7,\ -6,\ +14\ -26.$$

The sum of these divided by −880 gives a correction to the adopted coefficient of sin l of +0″·007.

The six sums of 8 of the Hansen series, periods 86–133, are

$$-1,\ -2,\ +10,\ -7,\ +1,\ -1,$$

giving a correction 0″·000. The maximum difference of the means of 8 from the general mean is 0″·13.

18. *The Terms with Period approximating to* 8*P.*—Since this analysis leads to the establishment of an empirical term, *i.e.* one which needs explanation from a source not hitherto considered, it seems advisable to give the evidence for its existence with some detail.

An analysis was made for a motion of 45° in a period. The change of phase tending to show that the period was the same within the limits of error as that of the perigee, the appropriate

longitude of the perigee plus a small correction to account for the fact that its period is not exactly 8P (the motion of the perigee in one period is 46°·13) was subtracted from each phase, so as to give the same results as would have been obtained from a direct analysis. The reason for this procedure is the ease and security with which an analysis for an integral division of the circumference can be performed.

For the coefficient of cos l, the term assumed is

$$\tfrac{1}{4}\kappa \cos\{\phi + 45^\circ(\mathrm{P}-1)\},$$

and for that of sin l,

$$-\tfrac{1}{4}\kappa' \sin\{\phi' + 45^\circ(\mathrm{P}-1)\},$$

where P is the number of the period.

In setting down the results for the Hansen series it is convenient to assume the result of the investigation and add 138° instead of 135° to the phase actually found, in order to account for the fact that in this series we start with period 86 instead of period 89. The following table gives the results of the analysis. In the eighth column is shown the value of ϖ at the beginning of each cycle of 8 periods plus the difference of its mean change from 360° over the cycle; in the Hansen series 138° has been added to each value of ϕ, ϕ'. The values of κ, κ' are derived in all cases from the values of $\kappa\cos\phi$, $\kappa\sin\phi$, $\kappa'\cos\phi'$, $\kappa'\sin\phi'$ in the same line; they require a slight adjustment on account of the motions of ϕ, ϕ'.

TABLE VI.

Analysis for Term of Period 8P. Units of 0″·1 *for* κ, κ'.

Per.	$\kappa\cos\phi$.	$\kappa\sin\phi$.	$\kappa'\cos\phi'$.	$\kappa'\sin\phi'$.	ϕ.	ϕ'.	ϖ.	κ.	κ'.
					°	°	°		
1–8	+ 4	+26	+29	+38	81	53	47	26	48
9–16	+53	+ 9	+11	+ 3	10	15	56	54	11
17–24	− 5	+13	+16	+17	111	47	65	14	23
25–32	+ 2	−43	−12	−20	273	239	74	43	23
33–40	− 7	+43	+ 6	−11	99	299	83	44	13
41–48	+ 8	+31	+17	+26	76	57	92	32	31
49–56	− 5	+32	+ 4	+ 9	99	66	101	32	10
57–64	+ 8	− 6	− 4	−12	323	252	110	10	13
65–72	+10	+10	+21	− 5	45	347	119	14	22
73–80	−27	+17	−20	+29	148	125	128	32	35
81–88	0	+18	+ 6	+19	90	72	137	18	20
86–93	+23	+ 7	+ 8	+ 2	156	152	146	24	8
94–101	+24	+11	+24	+ 7	163	154	155	26	25
102–109	+ 7	+ 9	+14	− 4	191	124	164	11	15
110–117	+14	+12	+15	+20	179	191	173	18	25
118–125	+19	+13	+12	+14	172	187	182	23	18
126–133	+11	+20	+16	+25	199	195	191	23	30
1–48	+ 9	+13	+11	+ 9	55	39	70	16	14
49–88	− 3	+14	+ 1	+ 8	102	83	119	14	8
86–133	+16	+12	+15	+11	175	174	169	20	19

The results for the individual periods of 8P in the Airy series tell little: in the Hansen series they are more consistent than might have been expected. The last three lines contain the group results. The differences of ϕ, ϕ', ϖ between the first two groups are $+47^\circ$, 44°, 49°, respectively; the mean change of ϕ, ϕ' is therefore practically equal to that of ϖ, but the mean phases are about 25° less. In going from the Airy series to the Hansen series the changes of phase of ϕ, ϕ' are greater than that of ϖ, but the actual phases in the Hansen series become nearly equal to those of ϖ. I argue from this that the periodic term in each case has the same period as ϖ, but that the observational corrections to the Airy series require some small additions depending on $\sin \alpha$, where α is the right ascension, and possibly on $\cos \alpha$ also. It is to be remembered that terms of period ϖ in the coefficients of $\cos l$, $\sin l$ will produce a term of period $l + \varpi = \alpha$ in the true longitude when the phases and amplitudes are equal.

19. From the last five columns it is now possible to form the analysis for a period ϖ: this is shown in the next table. It is of course unnecessary to set down the phases and amplitudes for the separate periods; the values of these for the group results are given at the end of the table.

TABLE VII.

Analysis for Term of Period ϖ. *Units of* $0''{\cdot}1$ *for* κ, κ'.

Per.	$\kappa \cos(\phi - \varpi)$.	$\kappa \sin(\phi - \varpi)$.	$\kappa' \cos(\phi' - \varpi)$.	$\kappa' \sin(\phi' - \varpi)$.
1–8	+22	+15	+48	+ 5
9–16	+38	−39	+ 8	− 7
17–24	+10	+10	+22	− 7
25–32	−41	−14	−22	+ 6
33–40	+42	+12	−11	− 8
41–48	+31	− 9	+25	−18
49–56	+32	− 1	+ 8	− 6
57–64	− 8	− 5	−10	+ 8
65–72	+ 4	−14	−15	−16
73–80	+30	+11	+35	− 2
81–88	+12	−13	+ 8	−18
86–93	+24	+ 4	+ 8	+ 1
94–101	+26	+ 4	+25	0
102–109	+10	+ 5	+11	−10
110–117	+18	+ 2	+24	+ 8
118–125	+23	− 4	+18	+ 2
126–133	+23	+ 4	+30	+ 2
1–48	+17	− 4	+12	− 5
49–88	+14	− 4	+ 5	− 7
86–133	+21	+ 3	+19	+ 1
	$\phi - \varpi$.	$\phi' - \varpi$.	κ.	κ'.
1–48	-13°	-23°	17	13
49–88	−17	−55	15	9
86–133	+ 6	+ 2	21	19

The group results are as consistent as can be expected from the separate periods. I shall assume that the period is that of ϖ. The phases and amplitudes of the coefficients of $\cos l$, $\sin l$ agree sufficiently nearly to enable us to treat their differences as accidental, and therefore to assume that they give a single term, argument $l + \varpi = \alpha$, in the true longitude. Hence, taking means, I deduce the observed corrections

$$-0''\cdot30\cos\alpha - 0''\cdot13\sin\alpha, \quad -0''\cdot50\cos\alpha + 0''\cdot05\sin\alpha,$$

to the Airy and Hansen series respectively. The probable errors of the coefficients are approximately $\pm 0''\cdot1$ and $0''\cdot05$ in the respective series. The results indicate that some small corrections, probably observational, are required to make the Airy series consistent with the Hansen series. Since the main question at issue is the satisfaction of the modern observations, I shall assume that the empirical term is $-0''\cdot50\cos\alpha$, and shall apply it to both series.

20. *Other Periodic Terms.*—As stated in No. 14, the 60P term, found in the coefficients of $\cos l$ before the application of $-2e\delta$L, has disappeared; no such term was found in the coefficients of $\sin l$ when the Fourier analysis was applied. The graphs also suggest a term of about three years, but an analysis of the Hansen period shows no evidence of it either in $\sin l$ or $\cos l$. It is, however, doubtful if a small term with this period could be detected from means extending over 414 days.

21. If we analyse the Hansen series for a term of period 16P, we obtain in the coefficient of $\cos l$ a term $+0''\cdot04\sin(22^\circ\cdot5t + 41^\circ)$ and in that of $\sin l$ a term $+0''\cdot17\sin(22^\circ\cdot5t + 96^\circ)$, where t represents the number of periods from P = 86. The former may be treated as accidental, but the latter is probably real. There are three cycles of the term in the series, and the three amplitudes are $+0''\cdot24$, $+0''\cdot14$, $+0''\cdot14$, while the phases are 93°, 85°, 114° respectively. It is not worth while to try the Airy series, owing to the large accidental errors present in it. The period cannot be more accurately determined from the three phases, though they indicate a period slightly shorter than 16P. The probable errors of the coefficients of $\sin 22^\circ\cdot5t$, $\cos 22^\circ\cdot5t$ are about $0''\cdot05$.

No other periodic terms have been found.

V. *Discussion of the Results.*

The Perigee.

22. The residuals have been analysed so as to obtain a correction to the adopted motion and position of the perigee. Two of the three constants found, however, are not independent of the other constants of the theory—namely, the mean motion and the secular acceleration. The question whether the observed and theoretical values can be made to agree naturally arises.

23. *The Secular Acceleration.*—As will be seen below, the observed value of this portion is $-35''\cdot7t_c^2$, while the theoretical

value is $-37''\cdot1t_c^2$. The difference, $1''\cdot4t_c^2$, between these is less than the possible error of the observed value. It therefore seems best to adopt the theoretical value, and add $-1''\cdot4t_c^2$ to the observed value. The epoch at which the change is made must be considered, since the adopted values of the position and mean motion will differ with different epochs. The date 1850 is near the weighted central date of the observations, and should therefore be chosen. If, then, we add $-1''\cdot4t_c^2$ to the formula at this date, we must add

$$-0''\cdot35+1''\cdot4t_c-1''\cdot4t_c^2$$

at the epoch 1800.

24. *The Mean Motion.*—This constant depends principally on the mean motion in longitude, the eccentricity, and the inclination —constants which are determined with an accuracy for this purpose much beyond that of the mean motion of the perigee. But it also depends on the action of the planets, and the figures of the Earth and Moon. The first of these admits of little doubt, but the portions arising from the two latter are not known to the degree of accuracy to which the perigee can be found from observation, because the compressions are not sufficiently well known. It therefore seems necessary at the present time to use the observed value of the perigee rather than its theoretical value determined with doubtful constants. Newcomb * found a value of the compression of the Earth which would reconcile the theoretical and observed mean motions of the perigee and node and the theoretical and observed coefficients of the periodic term in latitude with the mean longitude as argument. This may still be possible with the values found from the Greenwich meridian observations. But the observed motion of the perigee found here differs from his by $4''\cdot5t_c$, and he gave an erroneous value to the coefficient of the periodic term in latitude, supposed to be Hansen's. There are also other sources of error in the determination of this coefficient which I shall discuss in connection with the latitude.

25. *The Final Value.*—The adopted value of l is Hansen's (H) with three additions. The first (N) is Newcomb's addition to the mean longitude made in 1883, for which all the observations previous to that date were corrected by Cowell; the second (C) is an addition made by Cowell;† the third (Br) is the addition made by me to the mean motion. These are as follows:—

(H)	$110^\circ\ 19'\ 33''\cdot64+(1325^r+198^\circ\ 50'\ 37''\cdot15)t_c+49''\cdot435t_c^2$	
(N)	$-1''\cdot14$	$-29''\cdot17\,t_c-3''\cdot76\,t_c^2$
(C)	$+4''\cdot63$	$-13''\cdot99\,t_c+4''\cdot743t_c^2$
(Br)		$+1''\cdot93\,t_c$

To these must be added the observed correction found in No. 16 above:

$$+0''\cdot28+1''\cdot1t_c-5''\cdot2t_c^2.$$

* *Amer. Eph. Papers*, vol. ix. part i. p. 225.
† *M.N.*, vol. lxiv. p. 85.

The value of l found is therefore

$$110^\circ\ 19'\ 37''\cdot41 + (1325^r + 198^\circ\ 49'\ 57''\cdot02)t_c + 45''\cdot218t_c^2$$

at the epoch 1800.

The value of the mean longitude which was adopted in the corrections is* that which Newcomb obtained with some minor additions for neglected planetary terms, namely—

$$335^\circ\ 43'\ 25''\cdot56 + (1336^r + 307^\circ\ 53'\ 12''\cdot37)t_c + 9''\cdot52t_c^2.$$

Subtracting from this the expression for l, we obtain for ϖ the value—

$$225^\circ\ 23'\ 48''\cdot15 + (11^r + 109^\circ\ 3'\ 15''\cdot35)t_c - (35''\cdot70)t_c^2.$$

I have pointed out above that within the limits of error of the observations we can reduce the secular acceleration to its theoretical value, and that the addition at the epoch 1800 will be

$$-0''\cdot35 + 1''\cdot4t_c - 1''\cdot4t_c^2,$$

making the final observed value of the perigee with the theoretical value of the secular acceleration—

$$225^\circ\ 23'\ 47''\cdot80 + (11^r + 109^\circ\ 3'\ 16''\cdot75)t_c - 37''\cdot10t_c^2$$

at the epoch 1800·0.

I have omitted here the term in t_c^3 which Hansen uses, and which is obtainable from theory. It is entirely insensible in modern observations and produces a difference of about 1′ in the longitude of the Moon in 2500 years. It is a question whether it should not be neglected with other terms properly treated as insensible. It can only arise in the discussion of ancient eclipses, where quantities of the order of 36′ may be doubtful. It scarcely seems useful to complicate the formulæ of the tables with a minute addition which serves no useful purpose.

26. *Comparison with Theory.*—The sidereal value of the mean motion obtained above, reduced to the epoch 1850, is

$$1464353\overset{''}{6}t_c.$$

The theoretical value,† corrected to the compression 1/297 and to the values of the other constants which I have obtained, is

$$1464351\overset{''}{9}t_c.$$

The observed value is therefore $17''t_c$ greater than the theoretical. This is equivalent to a change of nearly 2″ in the coefficient of cos l in the course of a century. Such an error in the observed

* *M.N.*, vol. lxxiii. p. 701.
† *Mem., R.A.S.*, vol. lix. p. 94.

value is out of the question, as a glance at the graph of the residuals immediately shows, provided no error has been made in the corrections. I do not think that an error of this magnitude can have occurred in the theoretical computations, in view of the equations of verification which were computed. But, as stated above, the portions depending on the figures of the Earth and Moon are doubtful to about this amount, owing to the uncertainty of the adopted values of the compressions of those bodies. I shall discuss this in connection with a similar difference present in the mean motion of the node.

27. *Comparison with Other Determinations.*—Dr. Cowell found,* from the analysis of the same observations, the value $-33''t_c^2$ for the secular acceleration and $14643538''t_c$ for the mean motion. The differences are probably due to the terms of long period which I have added.

Newcomb adopts the theoretical value of the secular acceleration, making the change at 1800. In order to compare his value with mine, we must add to his result on this account $-0''\cdot35+1''\cdot4t_c$.

He has also neglected a planetary term of very long period which I have included. The correction to his value on this account is $+1''\cdot10+0''\cdot55t_c$. Adding these to his adopted value † I find for it,

$$225^\circ\ 3'\ 49''\cdot54+(11^r+109^\circ\ 3'\ 12''\cdot23)t_c-37''\cdot10t_c^2.$$

The difference Brown – Newcomb is therefore

$$-1''\cdot74+4''\cdot52t_c.$$

This is larger than the modern Greenwich meridian observations will admit, and a closer examination is therefore necessary in order to see whether an error of this magnitude is probable in the occultation results. The error is nearly three times the probable error given by Newcomb as proper to his determination.

28. Newcomb combines most of his results in groups of about eighteen years. I have taken these group results as given on p. 221 of his paper and added to them his observed correction of the perigee, thus obtaining his outstanding residuals: these are shown in the second column of the following table. The third column shows the residuals when they have been corrected to my value for the perigee — that is, Newcomb's outstanding residuals $+(-1''\cdot74+4''\cdot52t_c)$ ($\cdot11$). The last column shows the mean values of the outstanding residuals for approximately the same periods derived from the final columns of Tables III. (*a*), (*b*). My last value, epoch 1896, covers nine years only, while Newcomb's last value, epoch 1900, covers seventeen years.

* *M.N.*, vol. lxv. p. 275.
† *Loc. cit.*, p. 224.

TABLE VIII.

Comparison of Occultation and Meridian Results. Coefficients of cos l.

Mean Date.	Occ. (N).	Occ. (Br.).	Mer. (Br.).
	"	"	"
1720	−0·57	−1·16	...
1766	−0·36	−0·72	+0·10
1792	+0·74	+0·51	−0·25
1811	−0·03	+0·10	−0·25
1830	+0·24	+0·20	+0·09
1848	+0·19	+0·24	+0·07
1865	−0·16	−0·02	−0·01
1882	−0·19	+0·03	+0·08
1896	...	...	−0·08
1900	0·00	+0·31	...

The occultation results before 1800 are obviously subject to large errors.* After this date there is little to distinguish between the second and third columns; the only serious difference is in the last group. I have not been able to trace the source of the discrepancy. The immersions and emersions do not give essentially different results in the later groups. The comparison of the occultation and meridian residuals seems to show without much doubt the superiority of the meridian observations for the determination of the perigee. On this point Newcomb says (*loc. cit.*, p. 13), "Inequalities of short period can, as a general rule, be better determined from the meridian observations than from occultations. In this statement must be included the determination of the eccentricity and of the longitude of the perigee, taking for the latter its value at any recent epoch." On the whole it seems best to adopt the value obtained above.

The Eccentricity.

29. I give below in tabular form the various corrections made to the coefficient of sin *l*. The Airy and Hansen series are treated separately. The references are to volume and page of Dr. Cowell's papers in the *Monthly Notices* and to the numbered paragraphs of this paper.

	Airy.	Hansen.
Original coefficients	+22639".500 (lxiv. p. 572)	+22640".168 (lxiv. p. 163).
Cowell's additions	+0·200 (lxv. p. 268)	−0·548 (lxiv. p. 85).
		−0·200 (lxv. p. 268).
Corrections not yet applied (Nos. 9, 10).	−0·033	+0·032
	+0·001	+0·045
	−0·005	+0·009
	−0·128	+0·043
From analysis (No. 17)	+0·007	0·000
Observed coefficient	+22639·542	+22639·549

* On this subject see Nos. 22–27 of my longitude paper.

The probable error of the mean of these two results is about 0″·02. It is remarkable that they should agree so closely. It is convenient to neglect the last place in most investigations, so that the adopted value will be + 22639″·550.

Dr. Cowell, in his last investigation * of this constant, obtained 22639″·54, 22639″·46 for the respective series. The new corrections which have been added here account for the difference, 0″·08, between these values.

30. In the calculations, now in progress, necessary to construct the new lunar tables, the constant 22639″·500 has been adopted. But the form of the tables is such that any small change in this constant need not require the alteration of the actual tables, but only in certain added constants. In fact, until it is necessary to form the tables of arguments and to write the introduction, none of the six principal constants need be finally settled. And should any changes be required after the tables are published, they can be made with very simple instructions to the ephemeris computer. Further, the computations, though printed only to 0″·01, have been computed to 0″·001. Thus, should the number and the degree of accuracy of the observations increase so that any coefficient may be determined to 0″·001, it will be possible to compare the observed with the theoretical coefficient to this degree of accuracy.

The Empirical Terms.

31. *The Term with Right Ascension as Argument.*—It has been shown in No. 19 that there is a well-determined coefficient of 0″·5 to this term in the Hansen series, and that the coefficient in the Airy series is somewhat less, but that the difference may be attributed to incomplete observational corrections to the latter. There is only one theoretical term with which this term could be approximately identified—namely, one depending on $\sin(D + l') = -\cos(L - 10^\circ)$. The difference of phase is small, and may be accidental, although the evidence is against this assumption. But the theoretical coefficient of this term is well determined. Hansen's theoretical value is 18″·09, his tabular value 18″·17, Delaunay's value 18″·08, and mine 17″·99. These, reduced to the same value of the solar parallax, do not differ amongst themselves by more than 0″·06. A large error in the theory is therefore very improbable.

32. Thackeray † and Hough ‡ have brought out some evidence tending to show that Newcomb's Catalogue requires a correction depending on the right ascension α. The formula obtained by the latter after rejecting the terms with argument 2α, and expressing the results in seconds of arc, is

$$\Delta\alpha = +0''\cdot064\cos\alpha - 0''\cdot162\sin\alpha + (0''\cdot23\cos\alpha - 0''\cdot69\sin\alpha)t_c,$$

where t_c is reckoned from 1900. This gives a tabular − observed

* *M.N.*, vol. lxv. p. 745.
† *Ibid.*, vol. lxxi. pp. 174–178.
‡ *Ibid.*, vol. lxxiii. pp. 138–148.

correction, in units of 0″·025, and therefore proper for application to Table VI., of

$$-\Delta a = -3\cos a + 7\sin a + (-9\cos a + 28\sin a)t_c.$$

On the assumption that the only remaining correction should be of the form $a\cos a$, the evidence of Table VII. is against such a correction. Further, there appears to be no large change in the values of $\kappa\sin(\phi-\varpi)$, $\kappa'\sin(\phi'-\varpi')$ for periods 1–48 and 49–88, such as would be produced by the term $28t_c\sin a$; the same remark applies to the six separate periods in the Hansen series. In any case, the mean correction over the Hansen series is zero, so that the existence of the term $0''{\cdot}5\cos a$ is not affected by the assumption of this correction.

33. The term had been found by Dr. Cowell, but he had treated it as an error in the coefficient of $\sin(D-l')$. I think the evidence in No. 31 is conclusive against this explanation. I believe it is due to the irregular shape of the Moon's limbs near its Equator, as I stated at the end of my paper on the mean latitudes. This and other terms due to the same cause will be gathered together and discussed in my paper on the node and inclination.

34. *The Term, Period 16P.*—The period of the node is 16·4P, so that the term found in No. 21 may possibly be identified with an observed term, $+0''{\cdot}17\cos(\Omega-31^\circ)\sin l$. The origin of such a term is difficult to see. All the theories agree in making the coefficient of $\sin l$, due to the figure of the Earth, quite insensible, and I think that this supposition must be rejected. The hypothesis that some allied term has been neglected is also difficult, because in this case a corresponding term in the coefficient of $\cos l$ should be present. I have examined again the correction $-0''{\cdot}24\cos(2D-2l+2T-2T)\sin l$ which has been applied, but find no error in the tabulation; this term has a period of the right length.

Although the remark was made in No. 21 that the analysis of the Airy series was worth little, I carried it through from periods 6 to 85. The terms in $\cos l$ were again obviously accidental. In $\sin l$ the five cycles included gave phases 106°, 20°, 150°, 20°, −15° and amplitudes 0″·24, 0″·36, 0″·16, 0″·25, 0″·49 respectively; the phase for the mean was 27°, and the amplitude 0″·18. The amplitude is therefore the same as that deduced from the Hansen series, and the phase corresponds to a motion of 23°·6 per period; but I do not think that these results are entitled to much weight. The motion, however, is the same as that of the term with argument $2D-2l+2T-2J$ within 0°·2. In fact, it might be identified with an observed error $-0''{\cdot}18\cos(2D-2l+2T-2J+\phi)\sin l$, where ϕ is found to be −23°, but is uncertain by very nearly this amount.

The nature of the term is such that a theoretical cause for its existence is quite improbable, and I shall leave it aside as possibly due in some way to systematic irregularities of the limb.

35. *The Empirical Terms in the Mean Longitude.*—It has been shown that the empirical term (if existent as a *periodic* term) with

period 60–70 years and coefficient about 3″ is not present in either the perigee or the eccentricity. The residuals show the same fact about the great empirical term to the degree of accuracy of the results. The theory of the subject indicates that if the terms are due to a force similar to that of planetary action, they must in all probability arise from forces whose periods are a month or less. Inequalities of long period transmittted through their effect in the motion of the Earth (indirect) are much smaller in the motion of the Moon than in that of the Earth, and the effect on the perigee is of the same order of magnitude as that on the mean longitude. Forces whose periods are of the same order of magnitude as that of the perigee produce their greatest effects on the perigee and eccentricity. And forces of very long period require to be much larger, in order to produce a sensible effect, than those of short period which combine with a short-period term already present in the Moon's motion. We should therefore look for a small force of short period or a large force of long period to produce the observed effects in the mean longitude.

Yale University:
1914 *January* 13.

Correction of Errors in the New Lunar Theory.

By Ernest W. Brown, M.A., Sc.D., F.R.S.

The final results given on p. 96 of Part V. of my memoir on the theory of the motion of the Moon * should receive the following corrections:—

In the value of δw_2,

for "$+0''\cdot840\sin(w_3+276^\circ\cdot2)$" *read* "$-0''\cdot840\sin(w_3+276^\circ\cdot2)$";

for "+the ten periodic . . . w_1, w_2, w_3," *read* "$-\frac{5}{2}$(the ten periodic . . . w_1, w_2, w_3)";

and to the value of δw_3 add

"$+\frac{3}{4}$(the ten periodic terms in δw_1 whose angles are independent of w_1, w_2, w_3)."

These are additional to the corrections and additions given in *M.N.*, vol. lxx. pp. 3, 143.

These errors were not present on the computing sheets, but occurred in the collection of the results for publication. The first can be seen at once by a reference to the value of $\delta w_1 - \delta w_2$ given in p. 77 of the memoir. After discovering it, I found that Dr. Batterman had suggested in his memoir on the occultations of stars by the Moon † that it might be wrong. Nothing in my published work on the Moon's motion has been affected.

* *Mem., R.A.S.*, vol. lix.

† *Beob.-Erg. der k. Sternwarte zu Berlin*, No. 13 (1910), p. 18.

The Spectra of Hydrogen and Helium. By J. W. Nicholson, M.A., D.Sc., Professor of Mathematics in the University of London.

Dr. N. Bohr has recently given * a theory of spectra which, apart from its importance in pure physics, is of great significance to astrophysics, in that it defines, very precisely, some features of the state of the gases emitting bright line spectra in nebulæ and the early stars. We may summarise these features as follows:—

(1) According to the theory, an atom of hydrogen, when neutral, contains only one electron, and the ordinary Balmer spectrum arises during the formation of the atom from a positive nucleus, originally solitary, and an approaching electron.

(2) The atom of helium, when neutral, similarly contains two electrons, and the line $\lambda 4686$ of nebulæ and Wolf-Rayet stars, together with Pickering's series, hitherto ascribed to hydrogen, and the recent series obtained by Fowler, are due to helium with a single positive charge. They arise during the formation of a charged atom from a solitary positive nucleus and an approaching electron. Thus the spectrum can only be given by a gas when helium atoms are present which have lost all their electrons. If these lines are much stronger than ordinary helium lines, as is the case especially in the average Wolf-Rayet star, we must apparently suppose that the majority of the helium atoms are repeatedly in this condition. This leads to two alternative conclusions. Either there is a great scarcity of electrons in the nebulæ and in such stars, so that these bodies have, on the whole, a considerable positive charge, or electrical agencies are at work which are strong enough to separate the atoms completely into their positive and negative components. Either supposition would be of fundamental importance for the elucidation of the nature of these bodies, and also of the novæ.

(3) Some strong lines in the Wolf-Rayet spectrum, which can be arranged in a series analogous to Balmer's, and tending towards the same limit, are supposed to be due to lithium atoms, which are believed to have lost all their electrons, three in number, and to be on the point of taking back one electron. Again we have, even more definitely, the same alternative conclusions as in the case of the helium atoms, for there is no other evidence of the presence of lithium in these stars.

These results may, in the first place, be compared with phenomena manifested in the laboratory. This paper will subsequently examine the theory in more detail, for even in the non-dynamical form in which Bohr has given it, further development is easily carried on without the introduction of any new assumptions not already essential to the theory.

* *Phil. Mag.*, July 1913; see also *Monthly Notices*, Report of the Council, 1914.

As we have seen, two main possibilities exist—and apparently no others. Either these celestial bodies have a great preponderance of positive electricity, or they are in an electrical state which is powerful enough to tear all the atoms asunder. The first portion of the theory, relating to hydrogen, gives no information of value with regard to these points, for it is well known that the removal of an electron from a hydrogen atom is readily accomplished; and, moreover, if the Balmer spectrum is actually generated as the theory suggests, the readiness with which it appears in vacuum tubes is merely an indication of the weakness with which the electron is bound in the atom. As regards hydrogen, also, the theory suggests an explanation of a very puzzling fact,—that although, in the laboratory, it is difficult to obtain many members of the Balmer series, more than thirty can be detected in the chromospheric spectrum. According to Bohr, this is merely due to the tenuous state of the solar atmosphere, which allows the atoms to expand to a large multiple of their normal size. But apparently this tenuity must be much greater than in an ordinary vacuum tube, and we must bear in mind that such a supposition is contrary to evidence, mainly derived from displacements of spectral lines, tending to show that the chromosphere contains regions of very considerable pressure.*

It is instructive to compare Bohr's theoretical results for the helium atom with those obtained in the laboratory by Sir J. J. Thomson, with the help of his method of photographing the "positive rays" or positively charged atoms or molecules of a gas under electrical excitement. Thomson's experiments have given continuous evidence of the presence of helium atoms with one positive charge, but only rarely, and then in small quantity, with two charges. Such an atom with two charges would, on Bohr's view as well as Rutherford's, be nothing more nor less than an α particle, divested of the speed with which it leaves the atom of a radio-active body. It is apparently, therefore, difficult to remove two electrons from a helium atom; yet, according to the theory, Fowler, by obtaining the line λ 4686 strongly in his tubes, has succeeded in doing this very completely. The experiments have been repeated with success by Evans,† so far as the line λ 4686 is concerned, when hydrogen was apparently absent. If hydrogen was, in fact, spectroscopically absent, the experiments should have been pushed further, in an effort to obtain the line λ 5412 and other members of the Pickering series, for Bohr's theory requires the lines of the Pickering series to alternate in position with lines given very closely by Balmer's formula, whereas the intensity should change uniformly along the series. If λ 5412 and the next Pickering line were obtained without a line nearly coincident with H_β, the theory would be disproved.

* Evershed, however, in a recent Bulletin of the Kodaikánal Observatory, has, on the other hand, given serious reasons for doubting the existence of these pressures.

† *Nature*, Sept. 4, 1913.

Apart from this possible solution of the question, however, it is evident that a decisive test of the origin of the lines from helium might be obtained by Thomson's method. If a tube, while giving λ 4686 strongly, was examined by Thomson's method, and found to show positive ions, with an "electric atomic weight" equal to 2, more strongly than usual, Bohr's theory would be confirmed in a striking way. If they were not detected, the theory would have to be abandoned. It is very necessary that these crucial experiments should be performed. In the tubes used by Fowler, the theory requires these "positive rays" to be present in large numbers, and the relation of his experimental conditions to those of Thomson does not, at first sight, appear to support the view that they are present in the one case, and absent in the other.

We may proceed now to the theoretical side of Bohr's theory, relating to hydrogen and helium, in an endeavour to submit it to a more theoretical test. As already stated, it can be carried much further without any modification, or more precise dynamical specification.

The Spectrum of Hydrogen.

The Balmer formula does not give the main part of the ordinary terrestrial spectrum of hydrogen. In vacuum tubes, several hundreds of lines are obtained, constituting what is called the secondary spectrum, which is now recognised, after much controversy, as a true spectrum of hydrogen. Although the majority of these lines are weak, some are very strong, and of greater intensity in certain circumstances than H_γ and the later members of the Balmer series, which seems to be, in fact, the enhanced line spectrum of hydrogen. This is shown, for example, in a very striking manner in King's * recent experiments with the tube-arc, where the behaviour of the line H_α is exactly analogous to that of other well-known enhanced lines such as λ 4267 (carbon), which are prominent in the spectra of novæ and other celestial bodies. An examination of the secondary spectrum from the point of view of the Zeeman effect has been made by Dufour,† who, so far as the strongest lines are concerned, has divided it into two distinct parts,—one showing Zeeman effect, but not the other. This result is in accord with an investigation made by the writer from a purely numerical point of view, which shows that the stronger lines not giving a Zeeman effect have numerical relations more appropriate to a broken band than to a line spectrum. Some of these bands have actually been isolated experimentally by Fulcher,‡ who finds that they constitute the complete spectrum of hydrogen at very low voltages, where we may suppose that the molecules are not resolved into their constituent atoms. The view is now generally held that the secondary spectrum of hydrogen is a molecular spectrum,—not necessarily only that of

* *Astrop. J.*, 1913, Feb. and Nov.
† *Journal de Physique*, 1909, p. 258; *Ann. Chim. Phys.*, 1906, p. 416.
‡ *Astrop. J.*, 1913, Jan.

the molecule H_2, but perhaps also of such molecules as H_3, which Thomson has recently shown to exist.

The rest of the spectrum, following very different laws, and showing the Zeeman effect, is also supposed to be due to molecules. If it were due to atoms, it would be expected to occur in astronomical spectra. In this connection, it is interesting to note that its astronomical occurrence has been suspected, for example by Becker * in his analysis of the radiation from Nova Persei. In a certain type of Wolf-Rayet star,† moreover, Dr. Max Wolf found the spectrum to consist of the Balmer and Pickering lines, the line $\lambda 4688$, and a line $\lambda 4634$ which he characterises as a strong line of the secondary spectrum. This line, however, does not seem likely to have such an origin. It is one of the lines given by the generalised hydrogen formula suggested in the *Monthly Notices*, Feb. 1913, and the other lines of that formula, like the Pickering lines, do not correspond in any way with the secondary spectrum. If the origin of these lines suggested in a later paper ‡ is correct, they are due to an unusual state of the hydrogen atom peculiar to the Wolf-Rayet stars.

At the same time, however, the absence from astronomical spectra of that part of the secondary spectrum showing Zeeman effect, is no real indication that it is due to molecules. Sir J. J. Thomson has shown conclusively, in his series of papers on "Positive Rays," that large numbers of negatively charged hydrogen atoms are obtained in vacuum tubes. These atoms must have a spectrum, in whatever way spectra are produced, and we must conclude that some portion of the secondary spectrum of hydrogen arises from them. The absence of the secondary spectrum from astrophysics may therefore be taken to indicate the absence of negatively charged atoms in the celestial bodies whose spectra are known. Important conclusions follow. If we confine attention to the sun, for example, where hydrogen is very strong in the chromosphere, we find evidence that the chromosphere is on the whole positively charged. For if it were uncharged, then, since its radiation is strong, involving, on any theory of spectra, a continuous interchange of electrons among the atoms, there should be many negatively charged hydrogen atoms present at any instant. The fact that the chromosphere is probably endowed with a positive charge is significant, in view of the evidence deduced in former papers that the corona has a large negative charge. For it would seem that the corona consists of negatively charged atoms of protofluorine shot off, perhaps by a process of radio-activity or some process of a similar kind, from the chromospheric atoms. In such a case, they must leave a positive charge behind them in the chromosphere.

Returning to the question of terrestrial hydrogen, we must apparently suppose that its secondary spectrum is a mixture of the spectra of the molecular aggregates, and mainly of H_2, with

* *Trans. R.S.E.*, xli. 2. † *Sitzungs. der Heid. Akad.*, 1913.
‡ *Monthly Notices*, Jan. 1914.

those of charged atoms. Bohr's theory gives a concrete representation of the molecule H_2, and allows a calculation of its entire spectrum. We shall, in the first place, work out, according to the theory, the entire possible spectrum of hydrogen, corresponding to all the simpler stationary states of the atom or molecule, and compare it with observed data. For the atomic spectrum, Bohr has obtained Balmer's series and the Ritz series in the infra-red, as due to the various stages in the formation of a neutral atom from a nucleus $+e$ and an electron $-e$, each stage being characterised by a definite value of the angular momentum of the electron, equal to $\tau h/2\pi$ where h is Planck's constant and τ takes integral values.

If the electron is at any instant describing a circular orbit about a *stationary* nucleus νe, the angular velocity ω and radius a are related by

$$ma^3\omega^2 = \nu^2 e^2,$$

which merely expresses Kepler's law. If the orbit is elliptical, a denotes its semi-axis major and ω the mean angular velocity. For an angular momentum $\tau h/2\pi$,

$$2\pi m a^2 \omega = \tau h,$$

with the same extension to the elliptic orbit. The energy W radiated during the formation of the atom from a state of infinite dispersion is equal to the kinetic energy of the electron, or

$$W = \tfrac{1}{2} m a^2 \omega^2 = \frac{2\pi^2 m e^4 \nu^2}{h^2 \tau^2}.$$

If the atom had a ring of n electrons, we should find

$$W = \frac{2\pi^2 m e^4}{h^2 \tau^2} n(\nu - \tfrac{1}{4} S_n)^2$$

with the usual meaning of S_n.

For the passage between two stationary states with constant angular momenta $(\tau_1, \tau_2)h/2\pi$, Bohr's theory postulates the radiation of one quantum of energy, the product of h and the frequency, as a homogeneous beam. Thus the possible frequencies of the radiation are

$$\frac{2\pi^2 m e^4}{h^3}\left(\frac{1}{{\tau_1}^2} - \frac{1}{{\tau_2}^2}\right),$$

where τ_1 and τ_2 take all integral values. The Balmer formula is given by $\tau_1 = 2$, and the Ritz formula by $\tau_1 = 3$. Expressed in tenth-metres, the possible wave-lengths given in the formation of such an atom are

$$\lambda = \tfrac{1}{4}(3647{\cdot}14)\frac{{\tau_1}^2 {\tau_2}^2}{{\tau_2}^2 - {\tau_1}^2}.$$

$\tau_1 = 1$ gives lines in the Schumann region of the spectrum, which

is densely crowded with lines in the case of hydrogen. The corresponding series is

$$\lambda = 911{\cdot}785 \frac{\tau_2^2}{\tau_2^2 - 1}$$

whose members are

$$\lambda = 1215{\cdot}71,\ 1025{\cdot}76,\ \ldots\ .$$

Only the first line falls in the region considered in Lyman's paper.* He finds a line at λ 1216·0, which is the strongest in its immediate neighbourhood, being of intensity 8. If we can accept an error of 0·3 in the measurement, the series probably exists. If this be the case, hydrogen does actually show all the lines of wave-numbers

$$B\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$$

where B is Rydberg's constant, and m and n are integers.

Values of τ_1 beyond $\tau_1 = 3$ lead to series in the extreme infra-red. The above series contain, on Bohr's theory, the complete spectrum of the neutral atom of hydrogen. We may continue with the spectrum of a charged atom, containing two electrons, for hydrogen, as already pointed out, takes a negative charge fairly readily in Thomson's tubes. One of the electrons may be supposed to approach the atom, already containing the other electron, and we require, in the first place, to find all the possible stationary states or configurations in which the angular momenta can be maintained at specified values for a finite time, without the assumption of circular motion. For the purposes of stationary states, moreover, the theory equates mass-accelerations to *electrostatic* forces, the departure of the theory from ordinary dynamics resting, so far as steady states are concerned, with the forces calculated by ordinary electrodynamics on charges in motion. In the calculation of *steady motions*, forces of the latter type are to be ignored.

Suppose, therefore, two electrons and a nucleus in one plane, the electrons having constant angular momenta $(\alpha^2, \beta^2)h/2\pi$ about the common centre of gravity of the atom. As the nucleus is relatively heavy, having, in the case of hydrogen, about 1800 times the mass of an electron, we may neglect its motion, and suppose that it is continually at the centre of gravity. α and β should be the square roots of rational numbers.

If P and Q are the electrons, the nucleus attracts them radially, and as they repel each other with forces in the line PQ, they cannot move round the nucleus with constant angular momenta unless PQ passes through the nucleus continually. In other words, the angular velocities of the electrons are equal. If the radii of their orbits are r_1 and r_2 at any instant (the orbits are not necessarily circular), and if θ is the inclination of PQ to some

* Lyman, *Mem. Amer. Acad.*, vol. xiii., No. 3, p. 146.

fixed position, the steady orbits possible to the first electron are given by

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r_1}\right)+\frac{1}{r_1}=4\pi^2m^2r_1^2\frac{P_1}{a^4h^2},$$

where P_1 is the radial force towards the nucleus. Since the electrons are in a line,

$$P_1=\frac{\nu e^2}{r_1^2}-\frac{e^2}{(r_1+r_2)^2},$$

and therefore

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r_1}\right)+\frac{1}{r_1}=\frac{4\pi^2m^2e^2}{h^2a^4}\left(\nu-\frac{r_1^2}{(r_1+r_2)^2}\right).$$

Similarly for the second electron

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r_2}\right)+\frac{1}{r_2}=\frac{4\pi^2m^2e^2}{h^2\beta^4}\left(\nu-\frac{r_2^2}{(r_1+r_2)^2}\right).$$

But by the property of the angular momenta,

$$a^2r_2^2=\beta^2r_1^2,$$

and therefore the two equations for the orbits require

$$\beta^3\left(\nu-\frac{a^2}{(a+\beta)^2}\right)=a^3\left(\nu-\frac{\beta^2}{(a+\beta)^2}\right),$$

or

$$\nu(a^3-\beta^3)(a+\beta)^2=a^2\beta^2(a-\beta).$$

This is satisfied by $a=\beta$, so that the electrons are always equidistant from the nucleus. Any other possible value of $x=a/\beta$ must satisfy

$$\nu(x^2+x+1)(x+1)^2=x^2,$$

which has no real positive root, for any integral value of ν.

Thus even if the angular momenta are not integral multiples of $h/2\pi$, but have any possible constant values whatever, when the orbits of the two electrons are coplanar, there can be no stationary states except those in which the electrons are equidistant from the nucleus at every instant. The above analysis relates to the case in which the electrons are on opposite sides of the nucleus. If they are on the same side, the equation in a and β is

$$\beta^3\left\{\nu-\frac{a^2}{(a-\beta)^2}\right\}=a^3\left\{\nu+\frac{\beta^2}{(a-\beta)^2}\right\},$$

from which the same conclusion can be drawn.

Spectrum of the Neutral Hydrogen Atom.

Consider now more accurately the spectrum of a neutral hydrogen atom, in which both the nucleus and the electron rotate round the *common centre of gravity*, the angular momentum of the electron being $\tau h/2\pi$. If m and M are the masses of

electron and nucleus, a and b their distances from the centroid which is in a straight line with them, the conditions of steady motion are

$$ma\omega^2 = \mathrm{M}b\omega^2 = e^2/(a+b)^2$$

or

$$mw^2a^3 = e^2\left(1+\frac{m}{\mathrm{M}}\right)^{-2}.$$

The kinetic energy T and potential energy V are given by

$$2\mathrm{T} = \omega^2(ma^2 + \mathrm{M}b^2)$$

$$\mathrm{V} = \mathrm{C} - \frac{e^2}{a+b}$$

where C is the energy of the atom in a state of infinite dispersion.

Then

$$\mathrm{T} + \mathrm{V} = \mathrm{C} - \mathrm{T}$$

by the use of the equations of steady motion. In other words, the energy W which must be supplied in order to scatter the atom to infinity is equal to the total kinetic energy of the electron and nucleus. Bohr, in his paper, has not considered the motion of the nucleus, but evidently, when the nucleus does move, its kinetic energy must be included in W.

Thus

$$\mathrm{W} = \tfrac{1}{2}ma^2\omega^2\left(1+\frac{m}{\mathrm{M}}\right);$$

and since

$$ma^2\omega = \tau h/2\pi,$$

we have on reduction

$$\mathrm{W} = \frac{2\pi^2me^4}{\tau^2h^2}\left(1+\frac{m}{\mathrm{M}}\right)^{-3}.$$

If τ_1 and τ_2 are integers, the spectrum contains frequencies

$$\frac{2\pi^2me^4}{h^3}\left(\frac{1}{\tau_1^2}-\frac{1}{\tau_2^2}\right)\left(1+\frac{m}{\mathrm{M}}\right)^{-3}.$$

This must be identified, when $\tau_1 = 2$, with Balmer's formula. Any other spectrum whose frequencies are of the type

$$\alpha\left(\frac{1}{\tau_1^2}-\frac{1}{\tau_2^2}\right)\frac{2\pi^2me^4}{h^3}$$

therefore becomes, in tenth-metres,

$$\lambda = \frac{1}{4\alpha}(3647{\cdot}14)\frac{\tau_1^2\tau_2^2}{\tau_2^2-\tau_1^2}\left(1+\frac{m}{\mathrm{M}}\right)^{-3}.$$

Consider next the spectrum given by all the possible steady states of a hydrogen atom with a single negative charge, in which the electrons describe coplanar orbits. They are equidistant from the nucleus, and whether the orbits are circular or elliptical, the

spectrum is the same. The relation between angular velocity and radius is

$$\omega^2 = \frac{e^2}{ma^3}\frac{3}{4},$$

and the energy radiated in the collection of such an atom, when the angular momentum per electron is $\tau h/2\pi$, is

$$W = \frac{2\pi^2 me^4}{h^2\tau^2}\frac{9}{8}.$$

The corresponding spectrum of the atom, in its passage between pairs of such states, is

$$\lambda = 3647{\cdot}14\frac{\tau_1^2\tau_2^2}{\tau_2^2 - \tau_1^2}\frac{2}{9}\left(1 + \frac{m}{M}\right)^{-3},$$

and this formula should be accurate, for the positive nucleus does not move in these stationary configurations. If $\tau_1 = 2$, we get lines in the visible spectrum according to the formula

$$\lambda = 3236{\cdot}62\frac{\tau^2}{\tau^2 - 4},$$

where, for hydrogen, m/M has been written as 0·000544. The leading members of this series are

$$\lambda = 5825{\cdot}92,\ 4315{\cdot}49,\ 3853{\cdot}12,\ 3641{\cdot}20,\ \ldots$$

These lines are unknown in the secondary spectrum of hydrogen. The nearest recorded line is λ 5824·64, which is one of the weakest in the spectrum. The next is λ 5822·99, of intensity 3,* for which the difference is too large, as the theory necessitates an accuracy to five figures. The second line is distant two tenth-metres from the nearest line in the secondary spectrum, which is again a very weak one, of intensity 0. There is apparently no evidence for this series. We may conclude that if the theory is correct, hydrogen with a negative charge is not sufficiently stable to show a spectrum of its own. Yet this would contradict the experiments of Sir J. J. Thomson, already mentioned.

The angular momentum was taken, as in Bohr's theory in general, about the axis of the atom. If, for the case of the single electron, it is taken about the positive nucleus, the wave-lengths obtained are slightly different, but are still unknown in the spectrum.

It is interesting to attempt to modify the theory in another way. For if the angular momentum of a moving electron in the atom is always a multiple of $h/2\pi$, is the angular momentum of the nucleus, when it moves, also such a multiple, each angular momentum being taken about the common centre of gravity? This

* H. E. Watson, *Proc. Roy. Soc.*, 1909, A.

common centre must remain at rest, and the angular velocities of electron and nucleus about it must therefore be identical. If the angular momenta are $(\alpha^2, \beta^2)h/2\pi$, then, if (r_1, r_2) are the instantaneous radii of the orbits,

$$m(\ddot{r_1} - r_1\omega^2) = -\frac{e^2}{(r_1 + r_2)^2} = \mathrm{M}(\ddot{r_2} - r_2\omega^2)$$

with

$$mr_1^2 = \alpha^2 h/2\pi\omega, \quad \mathrm{M}r_2^2 = \beta^2 h/2\pi\omega,$$

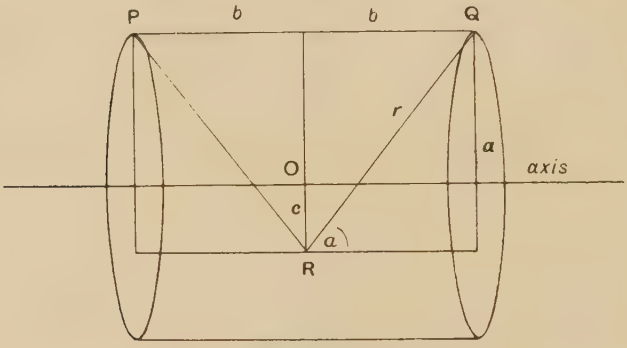
or

$$r_1\beta m^{\frac{1}{2}} = r_2\alpha \mathrm{M}^{\frac{1}{2}}, \quad r_1 m = r_2 \mathrm{M},$$

so that

$$\frac{\alpha}{\beta} = \sqrt{\frac{m}{\mathrm{M}}}.$$

There is a constant ratio between the multiples in any stationary



state, and no further spectrum can be obtained by changing the angular momentum of a moving nucleus.

On Bohr's theory, therefore, a charged atom of hydrogen gives no known spectrum in passing between its simpler stationary states, and the neutral atom can also give no further spectrum. It does not follow that a known spectrum may not exist for the more complicated states of the charged atom, in which the electrons do not move in one plane.

This paper will only deal with the simplest cases of possible stationary states, when the orbits are not coplanar. But these cases are those which would be expected as the more normal states of such an atom. The simplest configuration of this nature, for an atom with two electrons, is shown in the figure, where O is the common centre of gravity of the electrons P and Q and the nucleus R. The whole atom rotates round an axis through O parallel to PQ with a constant angular velocity ω. Let $\mathrm{OR} = c$, the radius of the orbit of an electron being a, and $\mathrm{PQ} = 2b$.

The angle α is given by $r \sin \alpha = a + c$, $r \cos \alpha = b$, where r is the distance from either electron to the nucleus *ve.*

The equations of steady motion are

$$\frac{e^2}{4b^2} = \frac{bve^2}{r^3},$$

$$ma\omega^2 = \frac{ve^2}{r^3}(a+c) = \tfrac{1}{2}\mathrm{M}c\omega^2$$

or

$$2mc = \mathrm{M}c,$$

denoting that the centre of gravity is at rest. The first equation gives

$$\cos^3\alpha = 1/4v,$$

so that, for example, in a possible neutral helium atom, $\alpha = 60°$.

Again,

$$\omega^2 = \frac{ve^2}{ma^3}\sin^3\alpha\left(\frac{a}{a+c}\right)^2 = \frac{ve^2}{ma^3}\left(1+\frac{2m}{\mathrm{M}}\right)^{-2}\sin^3\alpha\,;$$

and if

$$ma^2\omega = \tau h/2\pi,$$

the whole energy in the configuration is

$$\mathrm{C} - \frac{2ve^2}{r} + \frac{e^2}{2b} + ma^2\omega^2 + \tfrac{1}{2}\mathrm{M}c^2\omega^2,$$

where C is the energy in a state of infinite dispersion. By virtue of the preceding equations, this reduces to

$$\mathrm{C} - (ma^2\omega^2 + \tfrac{1}{2}\mathrm{M}c^2\omega^2),$$

so that the energy radiated during the formation of the atom is as usual equal to the kinetic energy of all the moving charges, including the nucleus. This kinetic energy W may be reduced to the form

$$\mathrm{W} = \frac{4\pi^2me^4}{h^2\tau^2}v^2\sin^6\alpha\left(1+\frac{2m}{\mathrm{M}}\right)^{-3}.$$

In passing between stationary states of this character, the atom emits the frequencies

$$\frac{2\pi^2me^4}{h^3}\left(\frac{1}{\tau_1^2}-\frac{1}{\tau_2^2}\right)2v^2\sin^6\alpha\left(1+\frac{2m}{\mathrm{M}}\right)^{-3}.$$

Comparing with Balmer's formula, the wave-lengths are

$$\lambda = 3647{\cdot}14\,\frac{\tau_1^2\tau_2^2}{\tau_2^2-\tau_1^2}(8v^2\sin^6\alpha)^{-1}\left(1+\frac{2m}{\mathrm{M}}\right)^3\left(1+\frac{m}{\mathrm{M}_1}\right)^{-3},$$

where M_1 is the mass of a hydrogen atom and M of the present atom.

Consider now the spectrum of a charged atom of hydrogen, assuming this configuration. In this case, $v = 1$, $\cos^3\alpha = \frac{1}{4}$, $8\sin^6\alpha = 1{\cdot}75536$

$$\left(1+\frac{2m}{\mathrm{M}}\right)^3\left(1+\frac{m}{\mathrm{M}}\right)^{-3} = \left(1+\frac{m}{\mathrm{M}}\right)^3 = 1{\cdot}00163$$

to a sufficient approximation, and we obtain

$$\lambda = 2081{\cdot}10\,\frac{\tau_1^2\tau_2^2}{\tau_2^2-\tau_1^2}.$$

With $\tau_1 = 1$ the series gives

$$\lambda = 2774{\cdot}80,\ 2341{\cdot}24,\ 2219{\cdot}84,\ 2167{\cdot}81,\ \ldots$$

With higher values of τ_1 we get lines beyond λ 8000, where only the Paschen or Ritz series is known.

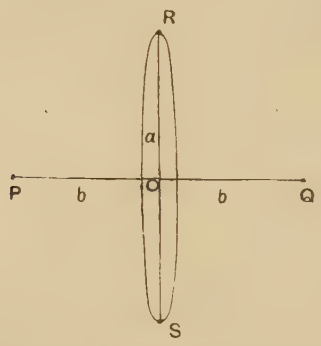
No line is recorded within thirty tenth-metres of the first member of this series, and the other members are also unknown. The series is completely unrepresented, so that the extension of the investigation to non-coplanar orbits of the simplest type gives no further justification to Bohr's theory.

We must suppose that either the theory is incorrect, or that negatively charged hydrogen does not show a spectrum. The second alternative is experimentally unlikely, and would greatly decrease the possibility of an ultimate explanation of the multitude of lines in the secondary hydrogen spectrum.

Spectrum of the Hydrogen Molecule.

If, therefore, the secondary spectrum of hydrogen cannot be derived from charged atoms, it must be due to the molecules. Bohr's theory does not deal with the portion of any spectrum which can be arranged in bands, but it leads to certain series spectra for the hydrogen molecule which will now be calculated.

The normal arrangement in this molecule consists of two



nuclei, of strength e, with two electrons rotating in a circle about the line joining the nuclei as axis. In the figure, P and Q are the nuclei, and R and S the electrons. $RO = a$, $PO = QO = b$.

The conditions of steady motion are

$$\frac{e^2}{4b^2} = \frac{2e^2 b}{(a^2+b^2)^{\frac{3}{2}}},$$

$$ma\omega^2 = -\frac{e^2}{4a^2} + \frac{2e^2 a}{(a^2+b^2)^{\frac{3}{2}}}.$$

From the first condition, $a = b\sqrt{3}$, and therefore

$$\omega^2 = \frac{e^2}{4ma^3}(3\sqrt{3}-1).$$

From the condition of angular momentum, $ma^2\omega = \tau h/2\pi$, and the energy radiated in the formation of such a molecule is

$$W = \frac{\pi^2 me^4}{4\tau^2 h^2}(3\sqrt{3} - 1)^2.$$

The resultant spectrum of the molecule, in its passage between pairs of such states, is, in tenth-metres,

$$\lambda = 3647{\cdot}14\ \frac{\tau_1{\tau_2}^2}{{\tau_2}^2 - {\tau_1}^2}\ \frac{2}{(3\sqrt{3} - 1)^2}\left(1 + \frac{m}{M}\right)^{-3}$$

$$= 413{\cdot}603\ \frac{{\tau_1}^2{\tau_2}^2}{{\tau_2}^2 - {\tau_1}^2}.$$

If $\tau_1 = 2$,

$$\lambda = 1654{\cdot}412\ \frac{\tau^2}{\tau^2 - 4}.$$

The leading lines are

$$\lambda = 2977{\cdot}942,\ 2205{\cdot}88.$$

The only recorded line near them is λ 2977·23, of intensity o. As this first line of the series should be the strongest line due to the molecule, we conclude that the calculated spectrum is not represented.

It is interesting to examine the possibility of the emission of the secondary spectrum of hydrogen during its passage between the molecular and atomic states. The energy radiated in the formation of the molecule directly from infinite dispersion to a configuration with angular momentum $\tau h/2\pi$ for each electron, is as above

$$W = \frac{\pi^2 me^4}{4\tau^2 h^2}(3\sqrt{3} - 1)^2.$$

The corresponding energy radiated in the formation of two atoms, each remote originally from the sphere of influence of the other, is

$$W_1 = \frac{2\pi^2 me^4}{h^2}\left(\frac{1}{{\tau_1}^2} + \frac{1}{{\tau_2}^2}\right)\left(1 + \frac{m}{M}\right)^{-3}$$

where τ_1 and τ_2 define their angular momenta.

If in a vacuum tube, energy is radiated in quanta during the passage from one state to the other, we should expect the system of lines of frequencies

$$\frac{2\pi^2 me^4}{h^3}\left\{\left(\frac{1}{{\tau_1}^2} + \frac{1}{{\tau_2}^2}\right)\left(1 + \frac{m}{M}\right)^{-3} - \frac{2}{\tau^2}\left(\frac{3\sqrt{3} - 1}{4}\right)^2\right\}$$

where τ, τ_1, τ_2 can take all integral values. The wave-lengths are

$$\lambda = 3647{\cdot}14\Big/\left\{4\left(\frac{1}{{\tau_1}^2} + \frac{1}{{\tau_2}^2}\right) - \frac{1}{2\tau^2}(3\sqrt{3} - 1)^2\left(1 + \frac{m}{M}\right)^3\right\}.$$

Isolating the special case $\tau_1 = \tau_2$, in which two atoms are alike, as being the normal case,

$$\lambda = \tfrac{1}{8}(3647{\cdot}14)\ \frac{{\tau_1}^2\tau^2}{\tau^2 - 1{\cdot}102305{\tau_1}^2}.$$

Wave-lengths in the visible spectrum are obtained for $\tau_1 = 2$, namely,

$$\lambda = 1823{\cdot}57 \frac{\tau^2}{\tau^2 - 4{\cdot}40922}$$

whose leading members are

$$\lambda = 3575{\cdot}02,\ 2517{\cdot}27.$$

No line has been found in the visible spectrum within thirty tenth-metres of either of these values. Further examination of the formula with τ_1 not equal to τ_2 leads to a similar negative result. The secondary spectrum cannot therefore be explained by the theory on any of the following grounds:—

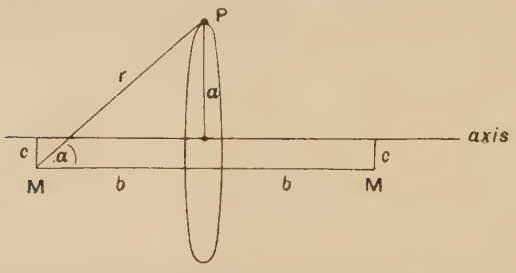
(1) That it is due to the atom, charged or uncharged.

(2) That it is the spectrum of the molecule, when neutral.

(3) That it is formed during the transition from the molecular to the atomic state.

The positively charged Molecule of Hydrogen.

A more significant test of the theory can be found in connection with the positively charged molecule of hydrogen. For this molecule, with a positive charge, is very conspicuous in Sir J. J. Thomson's experiments, and certainly ought to show a spectrum. Moreover, it contains only one electron, and it is only in cases



for which there is only one electron that Bohr's theory has been successful. The figure indicates the structure of such a molecule, —in fact, the only possible structure. The nuclei describe circles of radius c, and the electron a circle of radius a. The distance between the nuclei is $2b$. Then

$$ma\omega^2 = \frac{2e^2(a+c)}{r^3} = 2\mathrm{M}c\omega^2$$

$$\frac{e^2}{4b^2} = \frac{e^2 b}{r^3},$$

whence

$$\cos^3\alpha = \frac{1}{4},\ c = \frac{ma}{2\mathrm{M}},$$

and therefore

$$\omega^2 = \frac{2e^2}{r^3}\left(1 + \frac{m}{2\mathrm{M}}\right).$$

With the condition of angular momentum,

$$W = \frac{8\pi^2 me^4}{\tau^2 h^2} \sin^6 \alpha \left(1 + \frac{m}{2M}\right)^{-3}$$

on reduction, and the spectrum of this molecule becomes

$$\lambda = \frac{3647{\cdot}14}{16 \sin^6 \alpha} \frac{\tau_1^2 \tau_2^2}{\tau_2^2 - \tau_1^2} \left(1 + \frac{m}{2M}\right)^3 \left(1 + \frac{m}{M}\right)^{-3};$$

or finally

$$\lambda = 1038{\cdot}01 \frac{\tau_1^2 \tau_2^2}{\tau_2^2 - \tau_1^2}.$$

The case $\tau_1 = 1$ gives a Schumann series, whose members are

$\lambda = 1384{\cdot}01,\ 1167{\cdot}76,\ 1107{\cdot}21,\ 1081{\cdot}26,\ 1067{\cdot}66,\ 1059{\cdot}63.$

This gives no correspondence whatever with lines in the known Schumann spectrum, although all these members are in the region which has been examined carefully by Lyman,* whose results agree very closely with Schumann's. There is only one approximate coincidence with a strong line of that region, which is a region in which there is no obvious evidence for the existence of a series.

The case $\tau_1 = 2$ gives the lines

$\lambda = 7473{\cdot}67,\ 5536{\cdot}05,\ 4942{\cdot}90,\ 4671{\cdot}04,\ 4521{\cdot}11, \ldots$

The first was not found by Watson, there being apparently no lines in this part of the spectrum. The second is distant 1·8 tenth-metres from a measured line which was comparatively weak. The third is 2·1 tenth-metres from a very weak line. There is a possible coincidence in the case of the third, with a line of intensity 2 only, but the whole series fails to show any evidence of its existence as such. It should, as we have seen, be a strong series. Thus Bohr's theory cannot explain any portion of the hydrogen spectrum except the Balmer and Ritz series, and perhaps a Schumann series. It also predicts some strong series which are not found.

The Spectrum of Helium.

In passing to a consideration of the neutral helium atom, we may make use of the preceding results for two electrons, for the nucleus was taken with strength $2e$, including the case $\nu = 2$ which Bohr ascribes to helium. We have shown already that the only stationary states for two electrons in one plane are those in which the electrons are in line with the nucleus and equidistant from it. The energy radiated in the formation of such an atom is

$$W = \frac{2\pi^2 me^4}{\tau^2 h^2} 2(2 - \tfrac{1}{4})^2,$$

and in passing between pairs of such states the spectrum

$$\lambda = \frac{2}{49}(3647{\cdot}14) \frac{\tau_1^2 \tau_2^2}{\tau_2^2 - \tau_1^2} \left(1 + \frac{m}{M_1}\right)^3$$

should be emitted, where M_1 refers again to the hydrogen atom.

* *L.c.*

The Schumann spectrum of helium is unknown, so that the cases $\tau_1 = 1$, $\tau_1 = 2$ cannot be tested. $\tau_1 = 3$ gives

$$\lambda = 1341{\cdot}990 \frac{\tau^2}{\tau^2 - 9},$$

or

$$\lambda = 3067{\cdot}40,\ 2096{\cdot}86, \ldots$$

which are unknown.

$\tau_1 = 4$ gives

$$\lambda = 2385{\cdot}76 \frac{n^2}{n^2 - 16},$$

or

$$\lambda = 6627{\cdot}11,\ 4294{\cdot}35,\ 3542{\cdot}49 \ldots$$

which are unknown both in the terrestrial spectrum of helium and in astrophysical spectra. Thus the simplest stationary states produce an unknown spectrum. Passing to the states in which the orbits are in different planes, we may use the preceding analysis, given for negatively charged hydrogen, with $\nu = 2$, instead of $\nu = 1$. With the formula there derived, α is 60°, and the energy radiated in the formation of such a neutral helium atom is

$$W = \frac{2\pi^2 me^4}{h^2\tau^2} \frac{27}{8} \left(1 + \frac{2m}{M}\right)^{-3}.$$

Since M is four times the value for a hydrogen atom, we ultimately derive a spectrum

$$\lambda = 269{\cdot}9378 \frac{\tau_1^2\tau_2^2}{\tau_2^2 - \tau_1^2}.$$

When $\tau_1 = 3$ or 4, we get series in the visible spectrum, given by

$$\lambda = 5553{\cdot}01,\ 3796{\cdot}0,\ 3239{\cdot}25,\ 2976{\cdot}06$$

$$\lambda = 11997{\cdot}2,\ 7774{\cdot}2,\ 6413{\cdot}06,\ 5758{\cdot}67,\ 5382{\cdot}14,$$

which are quite unknown.

Nothing analogous to the ordinary helium spectrum can therefore be found, and a more complete investigation of *all* the stationary states is equally unsuccessful.

Other Aspects of the Theory.

It will be noticed that the cases considered in this paper all lead to expressions for the frequencies of the emitted radiation of the form

$$n = A\left(\frac{1}{\tau_1^2} - \frac{1}{\tau_2^2}\right)$$

where A is constant. This constant varies considerably, and though it always contains, as a factor, the Rydberg constant which Bohr identifies with $2\pi^2 me^4/h^3$, the other factors have no simple general form. In fact, Rydberg's constant, as a universal constant

of spectra, is not preserved. If we examine Bohr's deduction of the universal constant more closely, we find that his proof depends on the existence of stationary states in which one electron is approaching the atom, in a discontinuous manner, but with constant angular momenta which persist for a finite time. At the same time, the rest of the atom is behaving, effectively, like a single nucleus. We have seen that such stationary states do not exist, at least when the atom contains only two electrons. The results of another paper * indicate the absence of such states in other cases, and can be applied to prove its absence in all essential cases.

Such stationary states could not exist without a more complete overthrow of the ordinary dynamics than Bohr has suggested. The ordinary dynamics must fail even for the determination of stationary states.

If we nevertheless accept the ultimate validity of Bohr's theory, it is easy to see in what direction our dynamics must be modified further. For the theory is definitely successful when there is only one electron,—and also, at the same time, when there is only one nucleus, as we see from the failure in the case of the positively charged hydrogen molecule. The force between the single electron and the nucleus must still act according to the inverse square law, as Bohr supposes, but we must give up his further supposition that it continues to follow this law between two "bound" electrons, or between two nuclei in the same molecule. In this direction, but apparently in no other, the theory may be able to proceed, and some results which are outside the scope of the Society indicate that it can proceed, and preserve Rydberg's constant,—though at the same time other difficulties are raised. Even with these modifications, it does not seem possible to derive the ordinary helium spectrum.

Until some means of obtaining this spectrum from the theory is found, it rests on a slender foundation. For the most final test of an atomic model is its capacity to interpret spectra numerically. Thus we must conclude that at present the theory cannot throw much light on the astrophysical problems mentioned in the introductory paragraphs. For any further departure from the ordinary dynamics, except in the direction indicated, destroys the theory of the Balmer formula, and yet no laws of force between electrons appear to lead to the appropriate helium spectrum.

[*Note added* 1914 *March* 28.—In the discussion between Dr. Bohr and others in *Nature*, referred to in the paper, as to the exact wave-lengths of the Pickering series if due to helium, Dr. Bohr's calculation of the correction due to the nucleus, though details are not given, is different from that in the paper. The factor $(1+m/M)^3$ in the paper is replaced by $(1+m/M)$ itself. This can only occur if it is the *total* angular momentum in the atom which is related to Planck's h by a formula containing an integer—and not the angular momentum of each electron. In the discussion at the Society after the reading of the paper, Prof.

* *Phil. Mag.*, April 1914.

Eddington suggested that this procedure would give more latitude to the theory. Dr. Bohr has evidently used this method in obtaining the correction for the nucleus, though it is contrary to his general procedure. At the same time, however, though details need not be given, we find that no correspondence with the observed secondary hydrogen spectrum or the helium spectrum can be obtained with this method either, when applied to the preceding systems. It introduces only a difference of a few tenth-metres in some of the wave-lengths calculated above, but does not change the Schumann series for the neutral hydrogen atom.]

Micrometrical Measures of Double Stars in 1913.

By Rev. T. E. R. Phillips and H. F. Acocks.

Most of the stars in the following list have been measured with the 8-inch Coleman telescope lent by the Council of the Society. A few have been measured with the 12¼-inch Calver reflector.

The magnitudes have been taken from vol. lvi. of the Memoirs of the Society.

In the last column, A = H. F. Acocks, P = T. E. R. Phillips.

Star.	1910. R.A.	1910. Dec.	Mags.	Position Angle.	Dist.	No. of Nts.	Date.	Obs.
Σ	h m	° ′		°	″		1913	
60 (η Cassiopeiæ)	0 43·6	+57 20	4·0, 7·6	248·7	6·47	3	·711	P.
205 (γ Andromedæ A and B)	1 58·4	41 54	2·5, 5·5	114·7	0·63	4	·706	P.
228	2 8·2	47 4	6·7, 7·6	110·7	0·84	3	·751	P.
262 (ι Cassiopeiæ A and B)	2 21·6	67 0	4·2, 7·1	251·9	2·40	3	·754	P.
(A and C)			4·2, 8·1	110·7	7·48	3	·754	P.
535	4 18·3	11 10	6·7, 8·2	323·8	1·45	3	·929	P.
559	4 28·3	17 49	7·0, 7·0	278·7	2·98	3	·929	P.
982 (38 Geminorum)	6 49·6	13 18	5·4, 7·7	158·8	6·54	3	·148	A.
1126	7 35·3	5 27	7·2, 7·5	148·2	1·29	3	·017	P.
1348	9 19·8	6 44	7·5, 7·6	321·0	2·12	3	·232	P.
1355	9 22·5	6 38	7·2, 7·2	336·1	2·57	3	·226	P.
1356 (ω Leonis)	9 23·7	9 26	6·2, 7·0	122·8	0·96	3	·231	P.
1365	9 26·9	1 52	7·0, 8·0	158·2	3·50	3	·242	P.
1424 (γ Leonis)	10 15·0	20 18	2·0, 3·5	118·1	3·86	3	·172	P.
1439	10 25·1	21 16	8·0, 8·5	108·4	1·62	4	·261	P.

Star.	1910. R.A.	1910. Dec.	Mags.	Position Angle.	Dist.	No. of Nts.	Date.	Obs.
Σ	h m	° ′		°	″		1913	
1552 (90 Leonis)	11 30·0	17 18	6·0, 7·3	208·6	3·36	3	·290	A.
				210·8	3·24	3	·238	P.
1596 (2 Comæ Ber.)	11 59·7	21 58	6·0, 7·5	238·0	3·70	3	·263	P.
1663	12 32·7	21 42	7·8, 8·7	94·3	0·78	2	·301	A.
				94·7	·68	3	·245	P.
1687 (35 Comæ Ber.)	12 48·9	21 44	5·2 8·0	90·9	1·01	2	·270	A.
				89·8	1·09	3	·260	P.
1709	12 58·1	23 58	7·4, 9·7	249·3	2·55	3	·300	P.
1711	12 54·4	13 58	8·5, 9·0	348 6	0·78	3	·286	P.
1719	13 2·7	1 5	7·3, 7·8	0·92	7·13	3	·320	A.
				1·0	7·49	3	·273	P.
1734	13 16·1	3 25	7·2, 7·9	190·5	1·16	3	·274	P.
1742	13 19·8	1 52	7·4, 7·9	351·0	1·10	3	·282	P.
1777 (84 Virginis)	13 38·5	4 0	5·8, 8·2	230·6	3·15	3	·334	A.
				230·8	3·28	4	·319	P.
1781	13 41·6	5 34	7·8, 8·2	284·1	0·90	3	·353	A.
				284·7	1·00	3	·342	P.
1785	13 45·0	27 26	7·2, 7·5	334·0	1·35	4	·340	P.
1819	14 10·8	3 33	7·9, 8·0	346·3	1·31	3	·354	P.
1864 (π Bootis)	14 36·5	16 47	4·9, 6·0	103·0	5·91	3	·427	A.
1865 (ζ Bootis)	14 36·9	14 6	3·5, 3·9	140·2	0·80	3	·350	P.
1876	14 41·6	− 7 1	8·1, 8·6	80·8	1·24	3	·373	P.
1910	15 3·2	+ 9 35	7·0, 7·0	210·2	4·59	3	·362	A.
				210·8	4·56	3	·358	P.
1938 (μ^2 Bootis)	15 21·0	37 42	6·7, 7·3	55·1	1·26	3	·370	P.
1954 (δ Serpentis)	15 30·5	10 51	3·2, 4·1	182·6	3·85	3	·370	A.
				183·6	3·93	3	·365	P.
1965 (ζ Coronæ)	15 36·0	36 56	4·0, 5·9	304·4	6·13	3	·384	P.
2205	17 41·7	17 45	8·3, 8·7	311·9	1·58	3	·400	A.
2244	17 52·5	0 5	6·9, 7·1	276·5	0·96	3	·400	A.
				277·4	0·93	3	·384	P.
2245	17 52·5	18 21	7·0, 7·0	114·0	2·72	3	·397	A.
				113·7	2·73	3	·397	P.
2262 (τ Ophiuchi)	17 58·2	- 8 11	5·0, 5·7	259·2	2·14	3	·397	A.
				259·6	2·08	3	·397	P.
2264	17 57·7	+21 35	4·9, 4·9	260·4	6·38	3	·416	A.
				359·0	6·32	3	·391	P.
2272 (70 Ophiuchi)	18 1·0	2 32	4·5, 6·0	146·1	4·30	3	·504	A.
				145·0	4·20	4	·473	P.

Star. Σ	1910. R.A. h m	1910. Dec. ° ′	Mags.	Position Angle. °	Dist. ″	No. of Nts.	Date. 1913	Obs.
2339	18 29·8	17 39	7·2, 8·0	270·4	2·15	3	·426	P.
2351	18 33·3	41 13	7·4, 7·4	339·9	5·26	3	·422	P.
2382 (ϵ^1 Lyræ)	18 41·3	39 35	4·6, 6·3	12·0	3·33	3	·408	A.
				11·3	2·98	3	·404	P.
2383 (ϵ^2 Lyræ)	18 41·4	39 31	4·9, 5·2	121·9	2·52	3	·373	A.
				122·2	2·42	3	·406	P.
2443	19 0·0	14 40	8·2, 8·6	312·6	6·83	3	·433	A.
				312·1	6·83	3	·426	P.
2446	19 1·4	6 25	6·5, 8·4	153·9	9·84	3	·621	P.
2456	19 2·7	38 23	8·2, 8·2	183·1	23·16	1	·866	P.
2466	19 4·4	29 39	8·0, 8·5	103·9	2·50	3	·430	A.
				104 6	2·46	3	·428	P.
2524	19 22·9	25 20	8·3, 8·3	97·1	6·70	3	·809	P.
2525	19 22·9	27 9	7·5, 7·7	304·5	0·90	3	·496	A.
				304·0	0·85	3	·455	P.
2576	19 42·1	33 23	7·8, 7·8	283·9	2·09	3	·490	A.
				284·2	2·38	3	·455	P.
2589	19 48·0	0 27	8·0, 8·4	295·4	4·96	3	·680	A.
2609	19 55·3	27 52	7·0, 8·1	25·6	2·27	3	·817	P.
2624	20 0·1	35 47	7·2, 7·8	176·2	2·06	3	·455	P.
2626	20 0·7	30 17	8·0, 8·2	125·3	1·07	3	·461	P.
2644	20 8·0	0 36	7·1, 7·4	210·5	3·05	3	·455	P.
2723	20 40·6	11 59	7·3, 8·0	100·4	1·22	5	·542	P.
2744	20 58·5	1 10	6·3, 7·0	157·6	1·29	3	·648	P.
2767	21 6·4	19 36	8·1, 8·1	30·5	2·29	3	·581	A.
2779	21 10·5	28 42	8·5, 8·5	180·4	17·0	3	·883	P.
2895	22 17·0	24 29	8·5, 10·0	35·3	8·14	4	·638	A.
2909 (ξ Aquarii)	22 24·2	− 0 29	4·0, 4·1	308·0	2·90	3	·605	P.
2944	22 43·2	4 42	7·0, 7·5	260·9	3·23	4	·605	P.
2958	22 52·3	+11 16	7·2, 9·3	12·0	3·59	3	·581	A.
				13·0	3·61	3	·597	P.
3007	23 18·3	20 4	6·5, 9·5	82·7	6·35	3	·706	P.
3008	23 19·1	− 8 58	7·0, 8·0	328·3	3·49	3	·706	P.
3060	0 1·3	+17 34	8·7, 8·7	121·8	3·69	4	·714	P.

Observations of Occultations of Stars by the Moon, made at the Royal Observatory, Greenwich, in the Year 1913.

(*Communicated by the Astronomer Royal.*)

Day. 1913.		Phenomenon.	Mag.	Telescope.	G.M.T. h m s	Observer.
Jan. 12	Disapp.	B.D. −6°·6220	7·0	Sheepshanks	6 1 33·74	H.A.
18	,,	χ Tauri	5·7	Great Equatorial	9 3 40·38	H.F.
18	,,	χ Tauri (comes)	8·8	,,	9 4 1·22	H.F.
Mar. 23	Reapp.	Spica	1·2	Astrographic	9 22 15·98	W.
23	,,	,,	,,	Detached Tel.	,, 16·23	H.A.
May 10	Disapp.	B.A.C. 2383	6·5	Sheepshanks	10 15 6·21	H.A.
13	,,	34 Leonis	6·4	Detached Tel.	11 19 29·14	G.B.
13	,,	B.D. +14°·2203	8·9	Astrographic	9 1 21·68	W.S.
16	,,	Mayer 534	6·9	,,	8 15 28·50	W.A.L.
June 17	,.	B.A.C. 5737	6·7	,,	10 15 22·81	D.
July 20	,,	39 Aquarii	6·2	,,	12 48 52·81*	W.A.L.
Dec. 31	,,	ι Aquarii	4·4	,,	5 44 13·67	H.A.
31	,,	,,	,,	Detached Tel.	14·71	W.D.
31	,,	,,	,,	Sheepshanks	14·00	G.B.

The apertures of the telescopes used are as follows:—

Guiding Telescope of Astrographic Equatorial	10 inches.	Clock—	Dent 2014.
Sheepshanks Equatorial	6¾ ,,	,,	Earnshaw.
Great Equatorial		,,	Dent 2009.
Detached telescope in Great Equatorial Dome	4 ,,	,,	Dent 2009.
Detached telescope	4 ,,	,,	Loseby 111.

The initials D., H.F., W., W.S., H.A., W.D., G.B., W.A.L. are those of Mr. Dyson, Mr. Furner, Mr. Witchell, Mr. Stevens, Mr. Acton, Mr. Davies, Mr. Bartle and Mr. Lambert.

* Probably the star was lost to view before it reached the Moon's Limb.

On the Total Light of the Stars. By S. Chapman, B.A., D.Sc.

(*Communicated by the Astronomer Royal.*)

In a Memoir* by Mr. P. J. Melotte and myself, which was recently read before the Society, the results obtained from an extensive series of counts of stars, classified according to their Harvard photographic magnitudes, were collected and discussed; the number of stars of each photographic magnitude down to $17^{m}{\cdot}0$, in different galactic latitudes, was determined, and amongst the most important deductions may be mentioned the approximate constancy of the condensation of stars towards the galaxy, for stars of different magnitudes, and the great falling off in the rate of increase of $\log N_m$ per magnitude, for the faint stars. Here N_m denotes the total number of stars brighter than magnitude m, in the whole sky. The results indicated that N_m does not increase indefinitely with m, *i.e.* that the total number of the stars is finite; and it was found, moreover, that the observed values of N_m could be well represented by a formula from which an estimate of the total number of stars could readily be made.

The formula for N_m was

$$N_m = A\frac{1}{\sqrt{\pi}}\int_{-\infty}^{B(m-C)} e^{-x^2}dx, \tag{1}$$

and with regard to the constants A, B, C, it may be noted that A represents the total number of stars, and C the magnitude to which it is necessary to go in order to obtain half the total number of stars. A, B, C are connected with certain other constants a, b, c, occurring in the formula

$$\log \frac{dN_m}{dm} = a + bm - cm^2, \tag{2}$$

by the relations

$$\begin{cases} A = \sqrt{\dfrac{\pi \log_{10} e}{c}} \cdot 10^{a+\frac{b^2}{4c}} \\ B = \sqrt{\dfrac{c}{\log_{10} e}} \\ C = \dfrac{b}{2c}, \end{cases} \tag{3}$$

and a, b, c are easily determined from the observed data according to the equation (2).

Four different sets (I. to IV.) of values of a, b, c were deduced

* "The Numbers of Stars of each Photographic Magnitude down to $17^{m}{\cdot}0$, in Different Galactic Latitudes," by S. Chapman and P. J. Melotte, *Memoirs of the R.A.S.*, vol. lx. part iv.

from the observations, Sets II. and III. being regarded as the best, while I. and IV. were given to show the possible range of uncertainty in the constants deducible from the data, as any set of constants diverging from Set II., in either direction, further than do the Sets I. and IV., would introduce discrepancies between the calculated and observed values of $\frac{dN_m}{dm}$ which would exceed the probable error of the observations. These sets of values of a, b, c, together with the corresponding values of A, B, C, are as follows:—

	I.	II.	III.	IV.
c	0·0160	0·0150	0·0140	0·0130
b	0·720	0·700	0·680	0·660
a	−0·18	−0·09	+0·01	+0·09
C	22·5	23·3	24·3	25·4
A	760.10^6	1100.10^6	1800.10^6	3000.10^6
B	0·192	0·186	0·180	0·173

Of the last three constants the values of A are proportionately most uncertain, A being very sensitive to a small change in the constant c, while c is small and not very easy to determine with extreme accuracy; the three values of C are, on the other hand, not widely different one from another, and it may be said with fair certainty that half the stars are brighter than about the 24th magnitude.

The data of the Memoir may also be applied to the settlement of some points connected with the total light of the stars; this is the object of the present note. It is convenient to make the necessary calculations on the basis of the formula there given for N_m; for the present purpose, as will be seen, the uncertainty as to the exact number of very faint stars is of little consequence, the stars brighter than $17^{m}{\cdot}0$ being much the most important. For these the formula closely fits the observations, except in the case of the very bright stars, and a correction is applied on the latter account.

Let the intensity of the light of a star of zero magnitude be denoted by J. Then the intensity of light from a star of magnitude m is $J \cdot 10^{-0{\cdot}4m}$.

Hence the total intensity of light (dI_m) from all the stars ($\frac{dN_m}{dm}dm$ in number) whose magnitudes lie between m and $m+dm$ is

$$dI_m = J \cdot 10^{-0{\cdot}4} \frac{dN_m}{dm} dm$$

$$= J \cdot 10^{a+(b-0{\cdot}4)m-cm^2} dm$$

by equation (2). Hence, by a transformation similar to that used in the Memoir cited (p. 164), the following equation is obtained for

the combined intensity I_m of the light received from all the stars brighter than magnitude m :—

$$(4) \qquad I_m = J \,.\, A' \,.\, \frac{1}{\sqrt{\pi}} \int_{-\infty}^{B'(m-C')} e^{-x^2} dx$$

where

$$A' = \sqrt{\frac{\pi \log_{10} e}{c}} \,.\, 10^{a + \frac{(b-0\cdot4)^2}{4c}}$$

$$B' = B$$

$$C' = \frac{b - 0\cdot4}{2c}.$$

The physical significance of A′ and C′ is similar to that of A and C, viz. JA′ represents the intensity of the light from *all* the stars, while C′ is the magnitude to which it is necessary to go in order that the combined intensity of all stars brighter than this limit shall equal half that of all the stars. The four sets of values of a, b, c give the following values of A′, B′, C′ :—

	I.	II.	III.	IV.
A′	$10^{2\cdot380}$	$10^{2\cdot384}$	$10^{2\cdot399}$	$10^{2\cdot396}$
B′	0·192	0·186	0·180	0·173
C′	$10^{m}\cdot0$	$10^{m}\cdot0$	$10^{m}\cdot0$	$10^{m}\cdot0$

Hence we see that the total light received from all the stars is approximately of intensity

$$J \,.\, 10^{2\cdot40} = J \,.\, 10^{-0\cdot40} \,.\, 10^{2\cdot80}$$
$$= 631\, J \,.\, 10^{-0\cdot40}$$

But $J \,.\, 10^{-0\cdot40}$ is the intensity of the light of a 1st magnitude star. Hence, according to the formula, the total light of all the stars is equal to that of 631 stars of the 1st magnitude. This number, however, needs to be corrected to allow for the fact that the real number of very bright stars (up to $4^{m}\cdot0$ or $5^{m}\cdot0$) is in excess of that given by the formula for N_m—an excess which is unimportant in calculating N_m for most values of m, but which is very significant when the light of the stars is in question. The following table has therefore been made from the observed data of Table XV. of the Memoir; it gives the number of stars of magnitude 1·0, which would give an amount of light equal to that actually received from all the stars of magnitude m to $m+1$ (m ranging from 1 to 20). The three brightest stars have been given separately, and the other 1st magnitude stars have been treated individually; the last four lines of the table have been calculated from the formula II. Beyond the 6th or 7th magnitude the results given in the table agree with the calculated results, but the allowance for the bright stars brings up the total equivalent number of 1st magnitude stars to 690. The possible error of this result cannot be large. It is in error, if at all, most probably by

being too small, but it may be said with considerable certainty that the number is in the neighbourhood of 700, and can hardly be so much as 800.*

The Equivalent Light of the Stars.†

Magnitude.	Number.	Equivalent Number of 1st Magnitude Stars.	Totals to Magnitude *m*.
−1·6	Sirius	11	...
−0·9	α Carinæ	6	...
0·0	α Centauri	2	...
m m			
0·0–1·0	8	14	33
1·0–2·0	27	17	50
2·0–3·0	73	18	68
3·0–4·0	189	19	87
4·0–5·0	650	26	113
5·0–6·0	2,200	35	148
6·0–7·0	6,600	42	190
7·0–8·0	22,550	56	246
8·0–9·0	65,000	65	311
9·0–10·0	174,000	69	380
10·0–11·0	426,000	68	448
11·0–12·0	961,000	60	508
12·0–13·0	2,020,000	51	559
13·0–14·0	3,960,000	40	599
14·0–15·0	7,820,000	31	630
15·0–16·0	14,040,000	22	652
16·0–17·0	25,400,000	16	668
17·0–18·0	38,400,000	10	678
18·0–19·0	54,600,000	6	684
19·0–20·0	76,000,000	3	687
All stars fainter than $20^m{\cdot}0$		3	690

This number, it must be remembered, is stated in terms of 1st magnitude stars *on the photographic magnitude scale*. On the

* *Cf.* Newcomb, *Ap. J.*, xiv. 297; G. J. Burns, *Ap. J.*, xvi. 166; Kapteyn, *Groningen Publications*, No. 18. Also Newcomb, *The Stars*, chapter xviii. Newcomb's value, admittedly a rough one, is 800 stars of *zero* magnitude, or about 2000 stars of the 1st magnitude. Kapteyn's value is 2384 stars of the 1st magnitude on the Harvard visual scale. While the difference between the magnitude scales used by these writers and in the present paper renders a direct comparison difficult, it seems certain that these earlier values are too large.

† The numbers of stars in column 2 are taken directly from the table quoted; if the small irregularities in their rate of progression were smoothed out, the "run" of the numbers in column 3 would be improved, though with little effect on the final result.

visual scale the number would be nearer 900 or 1000,* on account of the fact that the photographic magnitude of the average star is numerically larger than the visual magnitude.

Our result for the total light of the stars, as being equal to that of about 700 1st magnitude stars, may be expressed otherwise: for instance, taking the magnitudes of Polaris and Sirius respectively as 2·1 and −1·6, the total light of the stars is equivalent to that of about 1750 stars as bright as Polaris, or about 60 as bright as Sirius.

Some further results from the formula (4) may be given here, though these also require small corrections on account of the brighter stars. From the value of C′ it is seen that the combined light of the stars brighter than $10^{m}\cdot0$ (or, from the table, $9^{m}\cdot5$) is equal to that of all the rest of the stars together. Again, by finding the values of m, which make the integral

$$\frac{1}{\sqrt{\pi}}\int_{-\infty}^{B'(m-C')} e^{-x^2}dx$$

equal to $\frac{3}{4}$, $\frac{7}{8}$, we may determine to what magnitudes it is necessary to go in order to obtain $\frac{3}{4}$ or $\frac{7}{8}$ of the total light of the stars. The corresponding values of $B'(m-C')$ are 0·477 and 0·814, and the values of m are as follows (the results for formula II. are alone given):—

	II.
m for $\frac{3}{4}$ of total light	12·5
$\frac{7}{8}$ „ „	14·3

These numbers also should be diminished slightly, for the same reason as before. They show how largely the total light which we receive from the stars is due to the stars brighter than $14^{m}\cdot0$, which are few in number compared with all the stars. This is due to the fact that beyond the 10th magnitude the stars do not increase in number (as we go to fainter limits) sufficiently to compensate for the diminution in brightness; this is especially noticeable round about the 17th magnitude.

As a final illustration of this, the proportion of the whole light of the stars which comes from the fainter half of their number will now be calculated. Half the stars are fainter than magnitude C of equation (1); the proportion of the total light received from this half of the stars is equal to

$$\frac{1}{\sqrt{\pi}}\int_{B(C-C')}^{\infty} e^{-x^2}dx = \frac{1}{\sqrt{\pi}}\int_{\frac{0\cdot2B}{C}}^{\infty} e^{-x^2}dx,$$

since

$$C - C' = \frac{b}{2c} - \frac{b-0\cdot4}{2c} = \frac{0\cdot2}{c}.$$

* I have found corroboration of this by calculations based on Prof. E. C. Pickering's figures for the total number of stars of each magnitude down to $13^{m}\cdot5$, given in *Harvard Annals*, xlviii. 149.

The value of this integral, corresponding to the constants of formula II., is

II.
0·00247.

Thus this half of the stars, several hundred millions in number, only afford about $\frac{1}{4}$ per cent. of the total light of the stars, an amount approximately equal to that received from four stars of the 2nd magnitude.

[*Note added later.*—It may be of interest to express the total light of the stars in terms of the light of the full moon and of the ordinary standard candles. This may be done by means of certain results due to King (*Harvard Annals*, 59, iii. and x.), who has given the value of the photographic magnitude of the full moon as $-11{\cdot}2$, and of a 2 candle-power electric lamp at 1 metre distance as $-12{\cdot}05$. Taking 700 as the number of $1^{m}{\cdot}0$ stars equivalent in total light to all the stars, it appears that the photographic magnitude of a single star equal in total light to all the stars would be $-6{\cdot}1$; the difference of 5 magnitudes between this and the full moon corresponds to a ratio of 100 in their light, *i.e.* the full moon gives 100 times as much light as all the stars together. Similarly it is found that the total star light is equal in photographic intensity to that of a 1 candle-power lamp at 10·8 metres (11·8 yards) distance, or to that of an ordinary 16 candle-power lamp at 47 yards distance.]

Baxendell's Observations of Variable Stars. No. 2, R Bootis; No. 3, R Cancri; No. 4, R Coronæ; No. 5, S Coronæ: with a correction to No. 1, R Arietis. Edited by H. H. Turner and Mary A. Blagg.

This is the second instalment of the publication of Baxendell's observations; the first being printed in *Mon. Not.*, lxxiii. p. 124. As applying to all the following stars, the list of telescopes used may be repeated here:—

A = 5-in. equatorial refractor, 70-in. focus, by Dancer.
R (7) = 7-in. reflector.
R (13) = 13-in. reflector, made by Baxendell.
R (12) = 12-in. reflector, belonging to Mr. Williamson.
30 (*a*) = 30-in. achromatic.
22 (*a*) = 22-in. achromatic, $2\frac{3}{4}$-in. aperture, by Dancer.
T = Small Tulley telescope, 35-in. focus.
C = Comet-seeker of $2\frac{1}{4}$-in. aperture.
E = 6-in. equatorial, $87\frac{1}{2}$-in. focus, used at Southport.

Fuller descriptions will be found in *M.N.*, lxvii. p. 316.

Baxendell's steps have been converted into tenths of magnitudes by means of the formula

$$t = 0{\cdot}915s - 0{\cdot}0115s^{2}$$

where s is the value in steps, and t in tenths. This was deduced by Miss Blagg from an extended discussion of the observations.

Note on No. 1, *R Arietis* (782 *Chandler*, 021024 *Harvard*).

$2^h\ 10^m\ 25^s\ +24^\circ\ 35'{\cdot}5$ (1900·0).

The observations of this star were printed in *Mon. Not.*, lxxiii. p. 124. Mr. C. L. Brook has since pointed out that the last four comparison stars have been wrongly identified. The identification was made by use of the figures on p. 153 of *H.A.*, xxxvii. (2), where the data are only given to $0^m{\cdot}1$ and 1′, which is scarcely sufficient for faint stars. On pp. 182 and 186, other figures are given which were previously overlooked and which make it clear that the identification should be corrected as follows:—

A.S.V.		H.A. Letter and Mag.			Adopted Mag.	
No.	Mag.	Old.	New.	Bax.	Old.	New.
18	10·7	$s=12{\cdot}87$	$r=12{\cdot}32$	x	125	126
19	10·8	...	$A=13{\cdot}19$	y	126	128
20	10·9	$r=12{\cdot}32$	$t=13{\cdot}26$	z	127	129
26	11·3	$t=13{\cdot}26$	not in *H.A.*	ϕ	135	137

It will be seen that the corrections, which have been deduced from the grades of the *A.S.V.*, would, at most, alter the inferred magnitudes by 0·2 near minimum.

No. 2, *R Bootis* (5237 *Chandler*, 143227 *Harvard*).

$14^h\ 32^m\ 47^s\ +27^\circ\ 10'{\cdot}2$ (1900·0).
$14\ \ 30\ \ 48\ \ +27\ \ 22{\cdot}1$ (1855·0).

Identification of Comparison Stars.—In this case Baxendell kept the same designations for the comparison stars throughout, as in the first column of Table I., except that x is twice used for l in 1866.

TABLE I.

Bax.	A.S.V.		Harv. Mag.	Adopted Mag.
	No.	Grad.		
a	1	...	...	76
b	2	0	8·44	84
c	3	6	8·80	91
d	4	10	9·05	95
e	7	30	10·25	103
f	9	36	10·58	106
g	10	40	10·80	108
h	12	44	11·01	110
p	13	47	11·18	112
k	16	59	11·63	116
m	20	77	12·34	123
l	—	...	(12·53)	125

The Harvard magnitudes are from *H.A.*, xxxvii. p. 194, with interpolations. The magnitudes adopted in the last column (decimal point omitted) have been estimated from Baxendell's differences as in Table II., taking the Harvard magnitudes as a guide.

TABLE II.

Baxendell's Differences between Comparison Stars. Unit 0·1 *Magnitude throughout. Subscript figures give numbers of Observations.*

Years.	84–76.	91–84.	95–91.	103–95.	106–103.	110–108.	116–110.
1860	12_6	...	...	...	...	...	...
1863	9_{13}	6_2	...	...	5_1	...	..
1864	8_3	16_6	...	12_2	...	6_1	8_2
1865–68	9_9	...	...	...	...	2_1	...
1878	9_8	...	...	...	4_1	8_1	...
1879	10_7	...	8_1	...	3_2	...	...
1880	8_8	8_2	6_1	11_2	6_1	5_1	8_1
1881–82	11_8	...	5_3	14_1	...	0_1	...
1884	13_8	...	...	...	...	3_2	...
1885–86	15_3	9_4	10_1	12_1	5_2	0_1	6_2
Means	10	11	6	12	4	3	7
No. Obs.	73	14	6	6	7	8	5
Reduced Means	7	8	4	9	3	2	5

Cases where the number of available comparisons does not exceed four have been omitted. It will be clear from the Means that Baxendell's steps are less than one-tenth. The simple sum of the means is 53, and of the differences at the heads of the columns 38. Taking one step as uniformly 0·072 mag., we should get the line of "reduced means," which agree fairly well with the heads of the columns. But an extended discussion of such cases showed that Baxendell's step was not uniform but could be expressed very closely (on the average) by the formula

$$t = 0{\cdot}915s - 0{\cdot}0115s^2$$

where s is the number of steps, and t the value in tenths; and this formula has been used to *whole tenths* in the reductions.

A list of the telescopes used is given in the first paragraph of this paper. The dates of use were as follows:—

A with power 68:—1859 April to 1868 July 24, with the exception of—

C with power 39 on 1863 April 30; May 8, 25, 26, June 12.

22 (*a*):—1859 April 8, 11, 16.

E with power 80:—1878 Feb. 20 to end, except on 1878 Sept. 13, when power 125 was used.

TABLE III.

Baxendell's Observations of R Bootis.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1860			
Apr. 8	0509	76 + 2, 84 − 6	78
8	0509	76 + 3, 84 − 7	78
11	0512	76 + 1, 84 − 8	76
16	0517	76 + 4, 84 − 9	78
20	0521	76 + 3	79
23	0524	76 + 2, 84 − 8	77
24	0525	76 + 2	78
25	0526	76 + 3, 84 − 8	78
May 1	0532	76 + 3	79
3	0534	76 + 3	79
7	0538	76 + 3	79
9	0540	76 + 3	79
1863			
Mar. 21	1586	84 + 3	87
26	1591	84 + 2	86
Apr. 1	1597	84 − 2	82
11	1607	76 + 0, 84 − 6	77
13	1609	76 − 1, 84 − 7	76
15	1611	76 + 0, 84 − 7	76
23	1619	76 + 0, 84 − 6	77
28	1624	76 + 0, 84 − 7	76
30	1626	76 + 0, 84 − 7	76
May 5	1631	76 + 1, 84 − 6	78
8	1634	76 + 2, 84 − 6	78
17	1643	76 + 2, 84 − 6	78
22	1648	76 + 3, 84 − 5	79
25	1651	76 + 6, 84 − 1	82
26	1652	76 + 8, 84 − 2	83
June 3	1660	76 + 7, 84 − 2	82
12	1669	84 + 6, 91 + 1	91
20	1677	91 + 3	94
23	1680	91 + 9, 106 − 8, 103 − 4	99
Aug. 29	1747	Invisible.	
1864			
Apr. 11	1973	116 + 2	118
12	1974	116 + 3	119
Apr. 25	1987	116 − 3	113
28	1990	110 + 1, 116 − 6	110
May 7	1999	108 + 1, 110 − 4	108
13	2005	95 + 3, 108 − 8	99
16	2008	95 + 3, 103 − 7, 108 − 9	98
24	2016	84 + 10, 91 − 2	92
26	2018	84 + 10, 91 − 2	92
29	2021	84 + 9, 91 − 3	90
31	2023	84 + 10, 91 − 3	90
June 6	2029	84 + 8, 91 − 5	89
8	2031	84 + 8, 91 − 5	89
16	2039	76 + 5, 84 − 3	81
July 4	2057	76 − 3	73
11	2064	76 − 4, 84 − 9	74
13	2066	76 − 3, 84 − 9	74
16	2069	76 − 2	74
19	2072	76 − 1	75
26	2079	76 − 1	75
29	2082	76 + 1	77
Aug. 21	2105	84 − 1	83
Sept. 24	2139	106 − 2, 108 − 3	104
Oct. 3	2148	108 + 6, 112 + 0	113
1865			
Mar. 30	2326	84 + 0	84
Apr. 10	2337	84 + 10	94
15	2342	84 + 11, 103 − 2	98
29	2356	112 + 0	112
May 1	2358	108 + 4, 112 + 0	112
18	2375	110 + 0	110
1866			
Feb. 14	2647	125 + 0	125
20	2653	125 − 2	123
Mar. 13	2674	108 + 0, 110 − 2	108
25	2686	106 − 3	103
Apr. 13	2705	84 + 0	84
21	2713	84 − 3, 76 + 8	82

TABLE III.—*Baxendell's Observations of R Bootis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1866			
Apr. 23	2715	84 − 2	82
29	2721	76 + 6, 84 − 3	82
May 7	2729	76 + 3, 84 − 5	79
15	2737	76 + 2, 84 − 6	78
22	2744	76 − 2, 84 − 7	76
24	2746	76 + 1, 84 − 6	78
June 8	2761	76 + 3, 84 − 2	80
25	2778	84 + 6	90
July 19	2802	103 − 3, 108 − 5	102
21	2804	103 − 2, 108 − 7	101
1867			
Feb. 28	3026	95 + 7, 106 − 7	100
Apr. 11	3068	116 + 0	116
27	3084	116 + 2	118
July 24	3172	76 − 3	73
27	3175	76 + 0	76
Aug. 1	3180	76 − 2	74
1868			
Apr. 2	3425	76 + 4, 84 − 4	80
9	3432	76 + 6, 84 − 3	82
23	3446	84 + 1	85
May 30	3483	108 + 4	112
1878			
Feb. 20	7036	84 + 1	85
Mar. 2	7046	84 + 4	88
13	7057	84 + 9, 103 − 2	97
23	7067	108 − 4	104
24	7068	108 − 5	103
Apr. 1	7076	106 + 4, 108 − 3	108
6	7081	108 + 4, 110 − 3	110
8	7083	110 + 0	110
May 5	7110	116 + 2	118
11	7116	116 + 3	119
21	7126	116 + 2	118
30	7135	116 + 1	117
June 23	7159	103 + 2, 106 − 2, 108 − 3	105
28	7164	103 − 1	102
1878			
July 7	7173	95 + 2	97
18	7184	84 + 1	85
20	7186	84 + 4, 95 − 4	90
28	7194	84 + 1	85
31	7197	84 − 1	83
Aug. 6	7203	76 + 7, 84 − 3	82
8	7205	84 − 4	80
10	7207	76 + 3, 84 − 4	80
14	7211	84 − 5	79
18	7215	76 + 3, 84 − 6	78
25	7222	76 + 3, 84 − 4	80
Sept. 9	7237	76 + 5, 84 − 5	80
11	7239	76 + 4, 84 − 3	80
13	7241	76 + 4, 84 − 3	80
24	7252	84 + 4, 76+10	87
Oct. 26	7284	108 − 3	105
1879			
Feb. 28	7409	84 + 1	85
Mar. 3	7412	76 + 6, 84 − 2	82
5	7414	76 + 7, 84 − 3	82
8	7417	76 + 7, 84 − 3	82
29	7438	76 + 2, 84 − 6	78
31	7440	76 + 2, 84 − 5	78
Apr. 6	7446	76 + 0, 84 − 5	78
Aug. 23	7585	116 + 6, 125 − 1	123
26	7588	116 + 6, 125 − 3	122
Sept. 12	7605	116 − 2	114
25	7618	103 − 2, 106 − 5	101
Oct. 6	7629	91 + 5, 95 − 2	94
15	7638	84 + 3	87
Nov. 1	7655	76 + 2, 84 − 9	76
1880			
Feb. 14	7760	110 + 5, 116 − 1	115
Mar. 21	7796	116 + 8	124
Apr. 19	7825	103 − 1, 106 − 6	101
24	7830	95 + 6, 103 − 3	100
May 1	7837	84 + 4	88
3	7839	84 + 4, 91 + 0	90

TABLE III.—*Baxendell's Observations of R Bootis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1880			
May 9	7845	84 + 1	85
24	7860	76 + 3, 84 − 3	80
29	7865	76 + 2, 84 − 5	78
June 4	7871	76 + 3, 84 − 3	80
7	7874	76 + 0, 84 − 6	77
17	7884	76 − 2, 84 − 8	75
July 4	7901	76 + 0, 84 − 7	76
9	7906	76 + 3, 84 − 5	79
20	7917	76 + 7, 84 − 3	82
31	7928	84 + 0	84
Aug. 6	7934	84 + 8, 91 − 1	91
9	7937	91 + 0	91
14	7942	91 + 3, 95 − 3	93
24	7952	95 + 7, 103 − 3	101
28	7956	103 − 2	101
Sept. 8	7967	108 − 1	107
16	7975	108 + 4, 110 + 0	111
28	7987	116 + 3	119
Oct. 3	7992	116 + 7, 125 − 1	124
1881			
Apr. 6	8177	95 + 9, 103 − 3	102
23	8194	108 + 3, 116 − 2	112
May 8	8209	116 + 2	118
20	8221	125 + 0	125
July 11	8273	108 − 6	102
27	8289	95 + 2, 91 + 6	97
Aug. 1	8294	91 + 3, 95 − 2	94
6	8299	91 − 1	90
24	8317	76 + 3, 84 − 4	80
30	8323	76 + 0, 84 − 6	77
Sept. 14	8338	76 + 3, 84 − 6	78
Oct. 16	8370	76 + 7, 84 − 2	82
29	8383	84 + 6	90
Nov. 5	8390	84 + 10	94
1882			
Mar. 12	8517	91 + 2, 95 − 1	94
Apr. 6	8542	76 + 4, 84 − 6	79
10	8546	76 + 6, 84 − 6	80
1882			
Apr. 20	8556	76 + 3, 84 − 8	78
May 16	8582	76 + 6, 84 − 3	82
July 24	8651	116 + 3	119
Sept. 13	8702	108 + 2, 110 + 3	112
1883			
Mar. 5	8875	116 − 1	115
12	8882	116 + 2	118
1884			
Feb. 25	9232	76 − 2, 84 − 8	75
Mar. 16	9252	76 + 6, 84 − 4	81
25	9261	76 + 8, 84 − 1	84
Apr. 7	9274	84 + 6	90
28	9295	108 − 3, 110 − 3	106
May 4	9301	108 + 3, 110 − 1	110
24	9321	116 + 3	119
30	9327	116 + 6, 123 + 1	123
June 25	9353	125 + 0	125
Aug. 19	9408	95 − 3	92
Sept. 12	9432	76 + 4, 84 − 9	78
17	9437	76 + 5, 84 − 3	81
20	9440	76 + 6, 84 − 7	80
Oct. 1	9451	76 + 3, 84 − 8	78
5	9455	76 + 1, 84 − 10	76
1885			
Mar. 11	9612	116 − 6, 110 − 2	109
27	9628	103 − 3, 106 − 6	100
Apr. 1	9633	103 − 3, 106 − 7	100
8	9640	84 + 8, 95 − 3, 91 + 6	94
17	9649	84 + 5, 91 − 3	88
23	9655	84 + 6, 91 − 4	88
29	9661	84 + 0, 91 − 9	83
May 7	9669	76 + 8, 84 − 5	82
20	9682	84 − 4	80
June 1	9694	76 + 10, 84 − 3	84
9	9702	76 + 9, 84 − 1	84
July 4	9727	95 + 8, 103 − 2	102
26	9749	110 + 2, 108 + 2	111

TABLE III.—*Baxendell's Observations of R. Bootis*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1885			
Aug. 14	9768	125 + 1, min. vis. − 6	126
Sept. 27	9812	116 + 6, min. vis. − 4	122
1886			
Mar. 3	9969	108 + 0	108
8	9974	106 + 3, 108 + 3	110
29	9995	116 + 2	118
1886			
Apr. 6	0003	116 + 4	120
26	0023	125 − 2	123
May 21	0048	110 − 1, 116 − 6	110
June 30	0088	84 + 3	87

The results of these observations are summarised in Table IV. according to the general procedure adopted in *Mem. R.A.S.*, vol. lv. for the Rousdon observations. But since the series are so much broken, it seems scarcely worth while to go beyond the tenths of a magnitude even in the means. Further, it seems doubtful whether the simple means should be taken, or the means weighted according to the number of observations, since the latter may give too much weight to an accidental irregularity. The former course is adopted in the means at the foot of the columns; but the latter has been also used in finding the correction to adopted epoch as below. The formula used is that of *Mem. R.A.S.*, vol. lv., except that instead of finding the correction to *period* from the differences of two rows, we first find the correction to *epoch* at the three mean dates *: viz. for

		Simple. d	Weighted d
I. + (13)	mean E = 12 approx. corr. to epoch =	+ 0·9	− 0·6
XXX. + (6)	,, E = 36 ,, ,,	+ 0·8	+ 0·7
XXXVII. + (6)	,, E = 43 ,, ,,	− 7·0	− 6·4

TABLE IV.

R Bootis. Assumed Period, 224 Days.

Group.	Portion. Julian Date 240+.	1. 0.	2. 19.	3. 38.	4. 56.	5. 75.	6. 94.	7. 112.	8. 131.	9. 150.	10. 168.	11. 187.	12. 206.
I.	0509	-2_4	-4_6	-7_2	...	...	...	...	...	...	...	...	...
V.	1405	...	...	...	...	...	...	...	...	...	...	$+2_3$	0_4
VI.	1629	-2_4	-2_4	-2_2	0_2	...	...	...	...	...	...	...	...
VII.	1853	...	...	...	...	...	...	-2_1	-1_3	-3_3	-4_5	-1_2	-3_3
VIII.	2077	-3_4	...	-2_1	-6_1	-2_1	...	...	...	...	...	...	...
IX.	2301	...	0_1	$+7_2$	$+7_2$	-5_1	...	...	...	...	...	...	...
X.	2525	...	...	...	...	...	...	...	$+6_2$	$+2_1$	$+4_1$	-1_2	$+2_3$
XI.	2749	-2_2	-1_1	$+5_1$	-1_2	...	...	...	...	...	...	...	...
XII.	2973	...	...	...	0_1	...	-4_1	-3_1	...	...	...	...	-3_3
XIV.	3421	0_1	$+1_2$	...	$+2_1$	...	...	...	...	..		.	...

* Since the following figures were obtained a few observations previously overlooked have been added; but the corrections to the figures below would be trifling.

TABLE IV.—*R Bootis. Assumed Period, 224 Days*—continued.

Group.	Portion. Julian Date 240+.	1. 0.	2. 19.	3. 38.	4. 56.	5. 75.	6. 94.	7. 112.	8. 131.	9. 150.	10. 168.	11. 187.	12. 206.
XXX.	7005	...	...	-2_2	-5_3	-6_3	...	-2_2	0_2	$+1_1$	$+1_2$	$+2_4$	$+2_5$
XXXI.	7229	$+1_2$	$+1_3$	...	$+2_1$	...	...	...	...	...	...	-2_4	$+1_2$
XXXII.	7453	0_1	...	...	...	...	...	...	$+9_2$	$+9_1$	$+6_2$	$+3_1$	0_1
XXXIII.	7677	...	...	...	...	-2_1	...	$+4_1$	...	-5_2	-11_3	-2_3	-2_2
XXXIV.	7901	-2_2	0_2	$+3_2$	0_2	-5_2	$+2_2$	...	...	...	...	...	...
XXXV.	8125	...	...	...	$+3_1$	-2_1	$+3_2$	...	...	-5_1	-1_3	0_1	0_2
XXXVI.	8349	...	0_1	$+3_2$	...	...	...	...	...	...	-2_1	0_1	$+1_2$
XXXVII.	8573	...	$+1_1$	...	...	$+2_1$	...	...	-4_1	...	...	...	...
XXXVIII.	8797	...	...	...	...	-2_1	0_1	...	...	...	...	...	...
XXXIX.	9021	...	...	...	...	...	...	...	...	...	...	...	-2_1
XL.	9245	$+1_1$	$+2_1$	$+5_1$	$+7_2$	$+4_2$	...	$+4_1$	...	...	-6_1	-1_3	0
XLI.	9469	...	...	...	...	...	...	...	...	0_1	0_3	$+3_3$	$+3_2$
XLII.	9693	$+4_1$	$+4_1$	$+15_1$	$+6_1$	$+10_1$	...	$+2_1$	...	...	...	...	...
XLIII.	9917	...	...	...	$+7_2$	$+1_1$	$+2_1$	$+2_1$	-6_1	...	-6_1	...	...
	Mag. from curve	79	82	89·5	105	115	120	121	115·5	106	95·5	83	77
I.	+(13)	−2	−1	0	0	−4	−4	−2	+2	0	0	0	−1
XXX.	+(6)	0	0	+1	0	−4	+2	+1	+4	0	−1	0	0
XXXVII.	+(6)	+2	+2	+10	+7	+3	+1	+3	−5	0	−4	+1	0

The epoch of maximum as estimated from the smooth curve used in forming the residuals (indicated by the mean magnitudes at the foot of the columns), falls about 205 days after (or 19 days before) the adopted zero, *i.e.* at the following dates:—

Curve Correction.	E.	Chandler.	Bax.−Ch.
2402301+205+0=2402506	12	2521·6−6·4	−9·2
7677+205+1 7883	36	7880·8+8·9	−6·7
9245+205−7 9443	43	9443·9+5·3	−6·2

Chandler's formula is

$$2399842 + 223{\cdot}3E + 9\sin(9^\circ E + 117^\circ),$$

the value of the periodic term being shown separately in the above comparison. The actual mean value of the difference, Baxendell-Chandler, is of little importance, as two independent estimations of the epoch of maximum from the same curve may easily differ by this amount. But the fact that the deviations from a mean value

are small, is satisfactory confirmation of Chandler's formula over the period covered by the observations.

Analysing the observations into harmonics we get—

$$\text{Adopted curve}\quad 2\cdot26\cos(\theta-168^\circ)+0\cdot03\cos 2(\theta-135^\circ)+0\cdot11\cos 3(\theta-120^\circ)$$
$$\text{E}=12\quad 2\cdot20\cos(\theta-170)+0\cdot13\cos 2(\theta-61)+0\cdot11\cos 3(\theta-97)$$
$$\text{E}=36\quad 2\cdot31\cos(\theta-170)+0\cdot13\cos 2(\theta-196)+0\,05\cos 3(\theta-94)$$
$$\text{E}=43\quad 2\cdot25\cos(\theta-157)+0\cdot12\cos 2(\theta-90)+0\cdot05\cos 3(\theta-72)$$

The second and third harmonics are so small that the phases are necessarily uncertain. But the discussion of these points is deferred for the present.

No. 3, R Cancri (2946 *Chandler*, 081112 *Harvard*).

$8^h\ 11^m\ 3^s\quad +12^\circ\ 2'\cdot0\ (1900\cdot0).$
$8\ \ 8\ \ 34\quad +12\ \ 10'\cdot1\ (1855\cdot0).$

The period being close to one year, a great part of the light curve is never observed. Hence the observations cannot be discussed in the usual way.

TABLE V.

Comparison Stars.

A.S.V. (II.) No.	A.S.V. (II.) Grad	Harv. Mag.	Baxendell. 1860 Feb. 12.	1862 Apr. 30.	1864 Apr. 21.	1868 Feb. 15.	Chart A. 1878 Dec. 7.	Adopted Mag.
...	...	...	...	...	...	...	2	74
...	...	...	...	...	...	...	10	75
2	8	7·55	*a*	...	7	...	7 and *z*	77
3	11	7·76	...	...	...	...	11	79
4	15	8·05	*b*	...	...	...	13	80
6	28	8·79	...	*a*	*a*	*a*	*a*	88
7	32	9·00	...	*b*	*b*	*b*	*b*	90
10	43	9·59	...	*l*	...	...	*y*	96
19	52	10·03	...	*n'*	*p*	...	*x*	100

The decimal point is omitted from the adopted magnitude in accordance with Pogson's convenient suggestion. Star 74 is B.D. $+11^\circ$ 1785 mag. 7·3, Potsdam (vol. 17) mag. = 7·50; Star 75 is B.D. $+11^\circ$ 1800 mag. 7·8. Both these are too far away to be included in Hagen's *Atlas Stell. Var.*

The Harvard magnitudes are from *H.A.*, xxxvii. p. 192, with interpolations.

TABLE VI.

Baxendell's Differences between Comparison Stars. Subscript figures give number of Observations.

Years.	75–74.	77–75.	80–77.	88–80.	90–88.	88–77.	100–96.
1857–59	...	$\cdot 13_6$	$\cdot 13_4$	$\cdot 88_3$	$\cdot 17_8$	..	$\cdot 12_3$
1860–62	...	$\cdot 21_4$	$\cdot 31_4$	$1\cdot 00_3$	$\cdot 30_2$	$\cdot 70_1$	$\cdot 30_1$
1863–64	...	$\cdot 22_8$	$\cdot 42_3$	$1\cdot 40_4$	...	...	.
1865–66	$\cdot 09_7$	$\cdot 28_8$	...	...	...	$1\cdot 80_1$	...
1868–70	...	...	...	...	...	...	$\cdot 20_1$
1878–79	...	...	$\cdot 13_2$	...	...	$\cdot 87_3$	...
1884	...	...	...	...	...	...	$\cdot 30_1$
Means	·09	·22	·25	1·12	·20	1·02	·19
No. Obs.	7	26	12	10	10	5	6

Cases where less than five comparisons are available have been omitted. The unit is ten times Baxendell's estimated tenth, or step, which is smaller than the standard tenth, and is apparently connected with it by the formula

$$t = 0\cdot 915s - 0\cdot 0115s^2$$

where s is the value in steps, and t in tenths.

A list of the telescopes used is given in the first paragraph of this paper. The dates of use were as follows:—

30 (*a*) or T from 1857 Jan. 23 to 1860 Mar. 18.

A 68 from 1861 Mar. 30 to 1870 Mar. 26. Except that C 39 was used from 1861 Jan. 1 to Mar. 8 and Apr. 9; 1862 Mar. 7, Dec. 26; 1863 Feb. 13 to 22; 1863 Nov. 4 to 1864 Feb. 5.

E 80 from 1878 Feb. 18 to end.

TABLE VII.

Baxendell's Observations of R Cancri.

Date.	J.D. 239+.	Corrected Comparisons.	Deduced Mag.	Date.	J.D. 239+.	Corrected Comparisons.	Deduced Mag.
1857				1857			
Jan. 23	9338	80 + 3	83	Mar. 16	9390	80 + 1	81
25	9340	80 + 3, 88 − 1	85	21	9395	80 + 1, 79 + 2	81
Feb. 12	9358	79 + 1, 80 − 3	78	24	9398	80 + 3, 96 − 3	88
24	9370	77 + 1, 79 − 2	78	Apr. 13	9418	88 − 1	87
25	9371	77 + 0, 79 − 1	78	16	9421	88 − 1	87
Mar. 4	9378	77 + 3, 79 + 1, 80 − 2	79	19	9424	88 + 1	89
				25	9430	88 + ?, 90 + 0	90
13	9387	79 + 1, 80 − 1	80	26	9431	90 + 1	91
15	9389	80 + 1	81	Nov. 15	9634	100 + 0, min. vis. − 4	100

TABLE VII.—*Baxendell's Observations of R Cancri*—continued.

Date.	J.D. 239+.	Corrected Comparisons.	Deduced Mag.
1857			
Nov. 18	9637	100 + 1	101
23	9642	96 + 0, 100 − 0	98
26	9645	100 − 2, 96 − 1	96
Dec. 7	9656	100 − 3, 96 − 2	96
20	9669	88 + 1, 90 + 0	90
23	9672	88 + 1, 90 + 0	90
1858			
Jan. 4	9684	88 − 1	87
9	9689	80 + 5, 88 − 2	86
16	9696	79 + 0, 80 − 1	79
20	9700	77 + 0, 79 − 1	78
Feb. 4	9715	75 + 0, 77 − 2	75
6	9717	77 − 3, 75 − 1	74
15	9726	77 − 3, 75 − 2	74
17	9728	77 − 2, 75 − 1	74
28	9739	77 − 2, 75 + 0	75
Mar. 3	9742	77 − 1	76
11	9750	77 + 0	77
15	9754	77 + 1, 80 + 0	79
16	9755	77 + 0, 80 − 1, 75 + 1, 79 + 0	78
21	9760	77 + 0, 80 − 1	78
26	9765	77 + 0, 80 − 2	78
Apr. 9	9779	80 + 3, 88 − 7	82
Nov. 10	9994	88 + 0	88
	240+		
28	0012	*80 + 3, 88 − 2	84
Dec. 2	0016	80 + 2, 88 − 4	83
1859			
Jan. 6	0051	77 − 2	75
28	0073	77 − 6	71
Mar. 18	0122	77 + 3, 79 + 2	80
Apr. 3	0138	88 − 1	87
1860			
Feb. 12	0453	77 + 2	79
16	0457	77 + 0, 80 − 3	77

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1860			
Feb. 16	0457	77 − 1, 80 − 3	76
20	0461	77 − 1, 80 − 3	76
Mar. 18	0488	77 + 3, 80 − 1	80
1861			
Jan. 1	0777	?80 − 9, ?79 − 8	?71
2	0778	?79 − 8, ?80 − 9, 77 − 4	73 or 72
3	0779	?79 − 8	?71
29	0805	75 − 7, 77 − 8	68
Feb. 6	0813	77 − 5	72
10	0817	77 − 5	72
24	0831	77 − 4	73
28	0835	77 + 1, 75 + 3	78
Mar. 8	0843	77 + 2, 75 + 4	79
30	0865	77 + 3, 88 − 3	82
Apr. 3	0869	80 + 3, 88 − 1	85
9	0875	80 + 9, 88 + 1, 90 − 1	89
May 4	0900	90 + 6, 100 − 10	93
1862			
Mar. 7	1207	80 + 9, 88 − 3	87
Apr. 30	1261	96 + 1, 100 − 1	98
Dec. 26	1501	77 − 3, 75 − 1	74
1863			
Jan. 23	1529	75 + 0, 77 − 3	74
Feb. 8	1545	75 + 3, 77 + 2, 80 − 2	78
12	1549	75 + 3, 80 − 3	78
13	1550	75 + 3, 77 + 3, 80 − 2	79
14	1551	77 + 3, 80 − 2	79
22	1559	80 + 6, 88 − 6	84
Mar. 3	1568	80 + 3, 88 − 7	82
15	1580	80 + 9, 88 − 3	87
21	1586	80 + 11, 88 − 1	89
Nov. 4	1814	88 − 1	87

* The figure is omitted in the original, and is supplied conjecturally.

TABLE VII.—*Baxendell's Observations of R Cancri*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1863			
Nov. 5	1815	88 − 2	86
8	1818	88 − 3	85
Dec. 2	1842	75 + 0, 77 − 2	75
17	1857	75 − 1, 77 − 3	74
1864			
Jan. 2	1873	75 + 2, 77 − 1	76
Feb. 5	1907	75 + 1, 77 − 1	76
8	1910	75 + 2, 77 + 0	77
12	1914	77 + 3	80
17	1919	77 + 2	79
Apr. 21	1983	90 + 6, 100 − 7	94
May 7	1999	90 + 8, 100 − 3	98
1865			
Jan. 4	2241	75 − 2, 77 − 4, 74 − 1	73
6	2243	75 − 2, 77 − 4, 74 − 1	73
20	2257	75 − 2, 74 − 1	73
Feb. 8	2276	74 + 1, 75 + 2	76
14	2282	75 + 2, 74 + 2	76
Mar. 1	2297	75 + 7, 77 + 3	81
9	2305	77 + 11, 88 − 3	86
30	2326	88 + 0	88
Dec. 14	2585	75 − 3, 77 − 5	72
26	2597	75 − 2, 74 − 1	73
1866			
Jan. 15	2617	75 + 3, 77 − 2	76
16	2618	75 + 2, 77 − 1	76
23	2625	75 + 2, 77 − 1	76
Feb. 13	2646	75 + 4, 77 + 3	80
20	2653	77 + 6, 88 − 3	84
Mar. 13	2674	88 − 1	87
1866			
Mar. 31	2692	96 − 4, 100 − 7	92
Dec. 7	2943	77 − 3, 75 − 2, 74 + 0	74
1867			
Jan. 2	2969	77 − 3	74
12	2979	77 + 0	77
1868			
Feb. 15	3378	88 − 2	86
Mar. 15	3407	90 + 4, 100 − 7	94
1869			
Feb. 5	3734	90 − 2	88
Mar. 2	3759	90 + 6	96
1870			
Mar. 26	4148	100 − 3, 96 − 2	96
1878			
Feb. 18	7034	96 − 4	92
Dec. 7	7326	77 + 0, 80 − 2	78
23	7342	80 + 1, 77 + 2	80
1879			
Jan. 16	7366	77 + 5, 88 − 1	84
22	7372	77 + 7, 88 − 1	86
25	7375	77 + 7, 88 + 0	86
Feb. 16	7397	88 + 6, 96 − 3	94
28	7409	?99 + 3, 100 − 2	100 or 98
1881			
Apr. 2	8173	?106 + 2	?108
1882			
Feb. 12	8489	96 − 8	88
1884			
Feb. 25	9232	96 − 2	94

TABLE VIII.

Analysis of the Observations.

Periods.	239+.	1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
I.	9300	...	82	79	84	89	...	...	..	...	...	...	98
II.	9660	88	76	75	79	...	...	...	...	...	...	...	85
III.	240+0020	...	73	...	84	...	...	...	...	...	...	...	...
IV.	0380	...	...	77	80	...	...	...	...	...	...	...	...
V.	0740	...	71	71	77	86	93	...	..	...	...	...	...
VI.	1100	...	...	...	87	...	98	...	...	...	...	...	...
VII.	1460	...	74	77	81	88	...	...	...	...	...	...	86
VIII.	1820	75	75	76	79	...	96	...	...	...	...	..	...
IX.	2180	...	...	73	78	87	...	...	..	...	...	...	...
X.	2540	...	72	76	82	87	...	...	...	..	...	...	...
XI.	2900	...	74	76	...	...	...	...	...	...	...	...	...
XII.	3260	...	...	...	86	94	...	.	...	...	...	...	...
XIII.	3620	...	...	...	88	96	...	...	...	..	...	...	...
XIV.	3980	...	...	...	...	96	...	...	...	...	...	...	...
XXI.	6860	...	...	...	...	...	92	...	...	...	...	...	...
XXII.	7220	...	...	...	78	82	88	(100)	...	...	...	...	...
XXIII.	7580	...	...	..	...	...	..	...	...	..	...	...	...
XXIV.	7940	...	...	...	...	...	...	...	(108)	...	...	...	...
XXV.	8300	...	...	...	...	...	...	88	...	...	...	...	...
XXVI.	8660	...	...	...	...	...	...	...	...	...	...	...	...
XXVII.	9020	...	...	...	...	...	...	...	94	...	...	...	...
Means		82	75	75	82	89	94	(94)	(101)	...	...	...	90
1st series	I.+(6)	88	75	76	82	88	95·5	...	...	...	...	...	90
2nd series	VIII.+(6)	75	74	75	83	91	96	..	..	...	...	...	...
3rd series	XXI.+(6)	...	...	...	78	82	90	(94)	(101)	...	...	...	...

No. 4, R Coronæ (5667 *Chandler*, 154428 *Harvard*).

$15^h\ 44^m\ 27^s$ $+28°\ 27'·8$ (1900·0).

15 42 36 $+28\ 36·3$ (1855·0).

Irregular.

1. *Comparison Stars.*—Identification proved very difficult owing to the number of sketches used; but ultimately all the puzzles were solved as below:—

TABLE IX.

Baxendell's Designations.							A.S.V.		H.A.	Adopted Mag.
Charts.			Names used.				No.	Grad.		
Undated.	1859.	1860–5.	1858.	1859–61.	1863.	1864.				
a (O)	*a*	*a*	O or *a*	*a*	*a*	*a*	1	0	7·18	72
b (*x*)	...	...	...	...	....	*b*	...	...	...	80
c (P)	...	...	P or *b*	*b*	...	*c*	...	...	...	79
d	...	...	*c*	*c*	...	*d*	2	27	...	91
e	*m*	*m*	...	*m*	...	*e*	7	46	10·29	97
f	*x*	*l* or *f*	*d*	*x*	*l*	*f*	15	72	10·66	106
g	*y*	*x*	*e*	*y*	?*x*	*g*	20	96	11·36	114
w	...	*w*	...	*z*	...	*w*	22	116	12·20	123
x	...	*x*	...	...	...	*x*	31	146	13·07	130
y	...	...	...	...	...	*y*	34	163	...	134
...	*n*	*n*	...	*n*	*n*	...	...	...	...	101
...	*p*	...	...	*p*	...	...	...	...	...	102
...	...	...	...	...	...	*π*	...	...	5·60	56

The magnitudes in the last column but one are from *Harvard Annals*, xxxvii., part 2, p. 168, except for π Coronæ, which is from the Revised Harvard Photometry (*H.A.*, vol. l.).

The telescopes used (see § 1 of this paper) were as follows:—

30 (*a*) from 1856 Sept. 4 to 1857 Sept. 10; 1857 Oct. 26 and Dec. 7; 1858 Jan. 20; Feb. 18; Mar. 11, 20, 26, 29; Apr. 1–4, 11; May 12, 30, 31; June 6 to 12, 15 to 27; July 28; Aug. 5, 8; Sept. 26.

T on 1856 Sept. 22 to Oct. 8; Oct. 28; Nov. 1, 6; 1858 Jan. 11; Feb. 1, 17, 24, 26; Mar. 6.

R (7) on 1858 Mar. 26; Apr. 4, 9, 12, 23; May 3, 21; June 4, 13, 18.

R (13) on 1858 Apr. 17, 20; May 1, 10, 19; Oct. 5; 1861 Apr. 13; 1865 Sept. 24; 1868 June 18; July 27.

A on 1858 May 5; July 30; Oct. 20; 1859 Apr. 17, 26; May 12–20, 23–27, 31; June 7–17, 22, 25, 29, 30; July 11, 13, 20, 27; Aug. 2, 19, 22, 27, 28; Oct. 3, 29; Nov. 3; 1860 June 18; July 8 to Oct. 14; 1861 Apr. 6 and May 14 to Nov. 15, except July 8; Aug. 16; (Mr. W.'s small achromated) and Sept. 1 (C); 1862 Mar. 3, 15, 31; Apr. 8, 10, 12, 29; May 16; July 28; Aug. 22 to Nov. 24, 1863 to 1869 inclusive, except dates for C mentioned below; R (13) on 1865 Sept. 24; 1868 June 18; July 27; 22 (*a*) on 1869 Mar. 6.

22 (*a*) on 1858 July 1–19; Aug. 9, 10, 21; Sept. 5, 17; Oct. 13, 20, 24, 28; Nov. 3, 6, 21; Dec. 5; 1859 Jan. 30 to Apr. 9; May 1–11, 21, 28, 29; June 20,

25; July 8–10, 15, 19; Aug. 18; Sept. 4, 10; Nov. 24; 1860 Feb. 6; June 6; June 27; Oct. 17 to Nov. 2; 1869 Mar. 6.

C on 1861 Feb. 6 to Mar. 30; Apr. 9, 16, 20; Sept. 1; 1862 Mar. 7; Apr. 4, 27; May 12, 19; June 16; July 4, 19, 24; Aug. 5, 16; Dec. 30; 1863 Feb. 13, 14, 28; Apr. 30; May 8, 25; June 12; July 17, 20, 26; Aug. 4, 17; Sept. 25, 28, 29; Oct. 30; 1864 Jan. 28; Feb. 5; 1865 Feb. 20; 1866 Jan. 18.

An opera-glass was used throughout 1877.

E was used from 1878 to the end, except on 1883 Apr. 3, May 4, when the opera-glass was used for comparisons with π Coronæ.

TABLE X.

Baxendell's Observations of R Coronæ.

Date.	J.D. 239+.	Corrected Comparisons.	Deduced Mag.
1849			
Mar. 16	6468	72 – 5	67
Apr. 1	6484	72 – 5	67
17	6500	72 – 5	
		Vis. to naked eye.	67
28	6511	Seen with naked eye.	
May 23	6536	72 – 5	67
July 1	6575	72 – 8	64
Aug. 16	6621	72 – 5	67
Sept. 11	6647	72 – 8	64
17	6653	72 – 8	64
Nov. 16	6713	72 – 8	64
1850			
July 4	6943	72 – 8	64
8	6947	72 – 8	64
28	6967	72 – 5	67
Sept. 7	7008	72 – 6	66
1851			
July 17	7321	72 – 5	67
Aug. 19	7354	72 – 5	67
Sept. 26	7392	Invisible.	
Nov. 17	7444	,,	
21	7448	,,	
1852			
Mar. 2	7550	,,	
8	7556	,,	

Date.	J.D. 239+.	Corrected Comparisons.	Deduced Mag.
1852			
Apr. 13	7592	*85 + 0	?85
19	7598	*85 + 0	?85
26	7605	*85 – , †85 + 0	?85
May 19	7628	72 + 1	73
24	7633	72 + 2	74
Aug. 2	7703	72 – 5	67
3	7704	72 – 4	68
13	7714	72 – 5	67
Sept. 1	7733	72 – 5	67
16	7748	72 – 8	64
1853			
May 2	7976	72 – 8	64
July 22	8057	72 – 8	64
1854			
July 23	8423	72 – 8	64
Sept. 10	8472	72 – 8	64
Oct. 15	8507	72 – 8	64
Nov. 8	8531	72 – 8	64
1855			
Apr. 21	8695	72 + 8	80
May 3	8707	Barely visible.	
7	8711	79 + , 80 + , 91 –	
8	8712	80 + , 91 –	
17	8721	80 + 8, 91 – 5	87

* This represents Lalande's 28930.

† This represents Lalande's 28977.

TABLE X.—*Baxendell's Observations of R Coronæ*—continued.

Date.	J.D. 239+.	Corrected Comparisons.	Deduced Mag.
1855			
June 23	8758	72 + 8, 79 + 0, 80 +	80
July 31	8796	Invisible.	
Aug. 2	8798	Suspected.	
24	8820	80 + 5, 91 − 8	84
30	8826	80 + 0	80
Sept. 5	8832	72 + 2, 79 − 5	74
20	8847	72 − 3	69
Oct. 5	8862	72 − 5	67
21	8878	72 − 8	64
1856			
Mar. 27	9036	72 − 8	64
May 4	9074	72 − 8	64
26	9096	72 − 8	64
July 30	9161	72 − 8	
		Vis. to naked eye.	64
Aug. 3	9165	72 − 8	64
6	9168	72 − 8	64
19	9181	72 − 8	64
Sept. 4	9197	72 − 8	64
7	9200	72 − 8	64
Oct. 1	9224	72 − 8	64
17	9240	72 − 8	64
Nov. 10	9264	72 − 8	64
Dec. 26	9310	Vis. to naked eye.	
1857			
Feb. 3	9349	72 − 8	64
25	9371	72 − 8	64
Apr. 19	9424	72 − 8	64
26	9431	72 − 8	64
May 4	9439	72 − 9	63
15	9450	72 − 7	65
19	9454	72 − 7	65
26	9461	72 − 8	64
28	9463	72 − 5	67
30	9465	72 − 6	66
June 12	9478	72 − 8	64
14	9480	72 − 8	64
1857			
June 17	9483	72 − 8	64
19	9485	72 − 7	65
22	9488	72 − 7	65
26	9492	72 − 7	65
July 1	9497	72 − 7	65
11	9507	72 − 8	64
Aug. 3	9530	72 − 8	64
9	9536	72 − 8	64
16	9543	72 − 7	65
27	9554	72 − 7	65
28	9555	72 − 7	65
Sept. 2	9560	72 − 7	65
10	9568	72 − 8	64
22	9580	72 − 7	65
28	9586	72 − 6	66
Oct. 4	9592	72 − 5	67
6	9594	72 − 3	69
8	9596	72 − 2	70
26	9614	72 − 3	69
28	9616	72 − 2	70
Nov. 1	9620	72 − 4	68
6	9625	72 − 6	66
Dec. 7	9656	72 − 8	64
1858			
Jan. 11	9691	72 − 7	65
20	9700	72 − 7	65
Feb. 1	9712	72 − 8	64
17	9728	72 − 6	66
18	9729	72 − 5	67
24	9735	72 − 6	66
26	9737	72 − 6	66
Mar. 6	9745	72 − 6	66
11	9750	72 − 6	66
20	9759	72 − 7	65
26	9765	72 − 3	69
26	9765	72 − 4	68
29	9768	72 + 0	72
Apr. 1	9771	72 + 1	73

TABLE X.—*Baxendell's Observations of R Coronæ*—continued.

Date.	J.D. 239+.	Corrected Comparisons.	Deduced Mag.
1858			
Apr. 1	9771	72 + 2	74
4	9774	72 + 2	74
4	9774	72 + 2	74
9	9779	72 + 7, 79 + 2	80
11	9781	72 + 9, 79 + 4, 91 − 3	84
12	9782	79 + 6, 91 − 4, 72 + 11	85
17	9787	91 + 2	93
20	9790	91 + 8, 106 − 2	102
23	9793	106 + 1	107
May 1	9801	91 + 2, 106 − 5	97
3	9803	91 − 2	89
5	9805	91 − 2, 106 − 10	92
10	9810	79 + 2, 91 − 5, 72 + 6	82
12	9812	72 + 7, 91 − 9, 79 + 0	80
19	9819	72 + 2	74
21	9821	72 + 1	73
30	9830	72 − 2	70
31	9831	72 − 1	71
June 4	9835	72 − 2	70
6	9837	72 − 2	70
7	9838	72 − 1	71
8	9839	72 − 2	70
12	9843	72 − 3	69
13	9844	72 − 3	69
15	9846	72 − 3	69
17	9848	72 − 3	69
20	9851	72 − 3	69
21	9852	72 − 4	68
24	9855	72 − 5	67
26	9857	72 − 6	66
27	9858	72 − 6	66
July 1	9862	72 − 5	67
6	9867	72 − 4	68
10	9871	72 − 4	68
1858			
July 17	9878	72 − 4	68
18	9879	72 − 4	68
18	9879	72 − 4	68
19	9880	72 − 4	68
28	9889	72 − 6	66
30	9891	72 − 5	67
Aug. 5	9897	72 − 5	67
8	9900	72 − 7	65
9	9901	72 − 6	66
10	9902	72 − 5	67
21	9913	72 − 7	65
Sept. 5	9928	72 − 7	65
17	9940	72 − 7	65
26	9949	72 − 8	66
Oct. 5	9958	72 − 4	68
13	9966	72 − 7	65
20	9973	72 − 9	63
20	9973	72 − 6	66
24	9977	72 − 9	63
28	9981	72 − 7	65
Nov. 3	9987	72 − 7	65
6	9990	72 − 7	65
	240+		
21	0005	72 − 9	63
Dec. 5	0019	72 − 9	63
1859			
Jan. 30	0075	72 − 9	63
Feb. 7	0083	72 − 9	63
23	0099	72 − 9	63
Mar. 7	0111	72 − 9	63
13	0117	72 − 8	64
18	0122	72 − 7	65
31	0135	91 − 1	90
Apr. 3	0138	91 + 4, 101 + 2, 97 + 2	99
4	0139	101 + 4, 97 + 4	103
6	0141	101 + 8, 106 + 0, 97 + 8	107

TABLE X.—*Baxendell's Observations of R Coronæ*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1859			
Apr. 9	0144	101 + 7, 97 + 7	106
17	0152	101 + 0, 106 − 2	102
26	0161	106 + 2	108
May 1	0166	106 + 4	110
2	0167	106 + 4	110
6	0171	106 + 4, 114 − 4	110
8	0173	106 + 5, 114 − 4	110
	(11·40)		
8	0173	Barely glimpsed.	
	(14·0)		
11	0176	106 − 1, 114 − 8	106
12	0177	106 − 2	104
13	0178	101 + 1, 106 − 4, 102 + 0	102
16	0181	101 − 1, 106 − 9	98
20	0185	101 + 0, 106 − 6	100
21	0186	102 + 0,	102
23	0188	101 + 3, 106 − 1	104
25	0190	101 + 6, 106 + 3	108
27	0192	106 + 6	112
28	0193	Invisible.	
29	0194	106 + 8	114
31	0196	114 − 1	113
June 7	0203	106 + 5, 123 − 3	116
9	0205	Suspected.	
10	0206	106 + 5	111
14	0210	106 + 4	110
15	0211	106 + 0	106
16	0212	101 + 2, 106 − 2	104
17	0213	101 + 0, 106 − 3	102
22	0218	91 + 8, 97 − 3	96
25	0221	91 + 2, 97 − 4, 106 − 12	93
25	0221	91 + 4	95
29	0225	91 + 2, 97 − 4	93
30	0226	91 + 2, 97 − 5	92
July 8	0234	91 + 4, 97 − 1	96
9	0235	91 + 6, 97 − 0	97
10	0236	91 + 6, 97 − 1	96
11	0237	91 + 5, 97 − 1	96
13	0239	97 + 0	97
15	0241	97 + 2	99
19	0245	97 + 0	97
20	0246	97 − 1, 91 + 5	96
27	0253	97 + 1	98
Aug. 2	0259	97 − 1	96
18	0275	91 − 6	85
19	0276	91 − 5	86
22	0279	91 − 5	86
Sept. 4	0292	72 + 3	75
10	0298	72 + 3, 79 − 3	76
27	0315	72 + 0	72
28	0316	72 − 2	70
Oct. 3	0321	72 − 2	70
29	0347	72 + 2	74
Nov. 3	0352	72 + 0	72
24	0373	72 − 4	68
1860			
Feb. 6	0447	72 − 5	67
12	0453	72 − 8	64
20	0461	72 − 7	65
Mar. 12	0482	72 − 11	61
14	0484	72 − 11	61
15	0485	72 − 11	61
24	0494	72 − 9	63
Apr. 1	0502	72 − 11	61
8	0509	72 − 11	61
11	0512	Visible to naked eye.	
16	0517	72 − 9	63
May 2	0533	72 − 8	64
13	0544	72 − 5	67
16	0547	72 − 7	65
29	0560	72 − 6	66
June 6	0568	72 − 6	66
18	0580	72 − 4	68
27	0589	72 − 8	64

TABLE X.—*Baxendell's Observations of R Coronæ*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1860			
July 8	0600	72 − 7	65
21	0613	72 − 6	66
29	0621	72 − 6	66
Aug. 31	0654	72 − 6	66
Sept. 22	0676	72 − 4	68
28	0682	72 − 5	67
Oct. 3	0687	72 − 6	66
14	0698	72 − 6	66
17	0701	72 − 6	66
20	0704	72 − 7	65
Nov. 2	0717	72 − 7	65
1861			
Feb. 6	0813	72 − 7	65
10	0817	72 − 8	64
24	0831	72 − 8	64
Mar. 8	0843	72 − 9	63
19	0854	72 − 9	63
30	0865	72 − 9	63
Apr. 6	0872	72 − 9	63
9	0875	72 − 10	62
13	0879	72 − 4	68
16	0882	72 − 8	64
20	0886	72 − 10	62
May 14	0910	72 − 8	64
17	0913	72 − 7	65
27	0923	72 − 7	65
29	0925	72 − 6	66
June 13	0940	72 − 8	64
16	0943	72 − 7	65
25	0952	72 − 6	66
30	0957	72 − 7	65
July 8	0965	72 − 6	66
26	0983	Invisible	<125
30	0987	,,	<127
Aug. 5	0993	,,	
16	1004	,,	<101
21	1009	,,	<124
28	1016	114 + 3	117
Aug. 30	1018	106 + 1, 114 − 4	108
Sept. 1	1020	97 − 1, 106 − 6	98
11	1030	72 + 0	72
12	1031	72 − 1	72
20	1039	72 − 3	69
Oct. 5	1054	72 − 4	68
9	1058	72 − 6	66
16	1065	72 − 6	66
28	1077	72 − 2	70
Nov. 2	1082	72 − 2	70
15	1095	72 − 4	68
1862			
Feb. 20	1192	72 + 1	73
Mar. 3	1203	72 − 2	70
7	1207	72 − 4	68
15	1215	72 − 4	68
31	1231	72 − 9	63
Apr. 4	1235	72 − 9	63
8	1239	72 − 9	63
10	1241	72 − 8	64
12	1243	72 − 9	63
27	1258	72 − 7	65
29	1260	72 − 7	65
May 12	1273	72 − 8	64
16	1277	72 − 7	65
19	1280	72 − 8	64
June 16	1308	72 − 7	65
July 4	1326	72 − 8	64
19	1341	72 − 6	66
24	1346	72 − 6	66
28	1350	72 − 6	66
Aug. 5	1358	72 − 7	65
16	1369	72 − 7	65
22	1375	72 − 6	66
Sept. 1	1385	72 − 6	66
10	1394	72 − 6	66
15	1399	72 − 6	66
18	1402	72 − 6	66

TABLE X.—*Baxendell's Observations of R Coronæ*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1862			
Sept. 20	1404	72 − 6	66
Oct. 4	1418	72 − 6	66
14	1428	72 − 6	66
18	1432	72 − 6	66
24	1438	72 − 6	66
29	1443	72 − 6	66
Nov. 24	1469	72 − 6	66
Dec. 30	1505	72 − 7	65
1863			
Feb. 8	1545	72 − 8	64
13	1550	72 − 9	63
14	1551	72 − 9	63
28	1565	72 − 9	63
Mar. 16	1581	72 − 8	64
21	1586	72 − 7	65
26	1591	72 − 8	64
Apr. 1	1597	72 − 8	64
11	1607	72 − 6	66
13	1609	72 − 6	66
15	1611	72 − 6	66
23	1619	72 − 5	67
28	1624	72 − 6	66
30	1626	72 − 7	65
May 5	1631	72 − 5	67
8	1634	72 − 6	66
17	1643	72 − 7	65
22	1648	72 − 7	65
25	1651	72 − 8	64
June 3	1660	72 − 7	65
12	1669	72 − 7	65
20	1677	72 − 5	67
23	1680	72 − 7	65
25	1682	72 − 6	66
30	1687	72 − 5	67
July 3	1690	72 − 5	67
17	1704	72 − 9	63
20	1707	72 − 8	64
26	1713	72 − 8	64
Aug. 4	1722	72 − 9	63
17	1735	72 − 5	67
24	1742	72 − 5	67
29	1747	72 − 5	67
Sept. 5	1754	72 − 6	66
20	1769	72 − 7	65
25	1774	72 − 9	63
28	1777	72 − 8	64
29	1778	72 − 8	64
Oct. 9	1788	72 − 6	66
11	1790	72 − 6	66
14	1793	72 − 6	66
30	1809	72 − 8	64
Nov. 2	1812	72 − 4	68
10	1820	72 − 1	71
29	1839	101 + 3, 106 − 3, 114 − 6	105
Dec. 17	1857	More than 106 + 8	
1864			
Jan. 28	1899	Not seen < 95	
Feb. 5	1907	,, < 100	
Mar. 28	1959	,, < 125	
Apr. 11	1973	,, < 130	
18	1980	,, < 110	
21	1983	Not seen < 120	
25	1987	,, < 130	
28	1990	130 + 0	130
30	1992	130 + 0	130
May 7	1999	130 + 2	132
13	2005	Not seen < 125	
16	2008	? Glimpsed < 125	
24	2016	Glimpsed = 128	
26	2018	,, = 130	
29	2021	Not seen < 127	
June 2	2025	Glimpsed = 132	
8	2031	Not seen < 115	
16	2039	,, < 117	
July 4	2057	123 − 3	120

TABLE X.—*Baxendell's Observations of R Coronæ*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1864			
July 8	2061	123 − 3	120
11	2064	123 − 3	120
13	2066	123 + 2	125
16	2069	Not seen < 110	
19	2072	,, < 115	
26	2079	130 + 3	133
29	2082	130 + 2	132
Aug. 21	2105	Not seen < 130	
23	2107	,, < 130	
26	2110	Suspected < 131	
Sept. 5	2120	Glimpsed 133	
11	2126	Invisible < 117	
22	2137	Glimpsed 132	
24	2139	130 + 3	133
Oct. 3	2148	130 + 2	132
31	2176	Glimpsed 133	
1865			
Feb. 20	2288	Not seen < 101 + 4	
Mar. 9	2305	Not seen < 101 + 4	
30	2326	106 − 3	103
Apr. 2	2329	106 + 0	106
8	2335	106 − 4	102
10	2337	106 + 0	106
15	2342	106 − 4	102
24	2351	97 + 2, 106 − 4	100
27	2354	97 + 4, 106 − 3	102
29	2356	97 + 3, 106 − 3	102
May 1	2358	97 + 2, 106 − 4	100
6	2363	97 − 1, 106 − 7	98
15	2372	91 + 2, 97 + 0, 106 − 8	96
18	2375	91 + 1, 97 − 1	94
20	2377	91 + 1, 97 − 1	94
24	2381	91 − 1	90
June 8	2396	91 − 2	89
11	2399	91 − 2	89
13	2401	91 − 2	89
17	2405	79 + 7, 91 − 3	87
1865			
June 20	2408	80 + 9, 91 − 3	88
23	2411	80 + 7, 91 − 3	88
28	2416	80 + 3, 91 − 3	86
July 1	2419	91 − 3	88
13	2431	80 + 4, 91 − 6	84
19	2437	80 + 3, 91 − 6	84
21	2439	80 + 3, 91 − 6	84
Aug. 3	2452	80 + 6, 91 − 3	87
8	2457	80 + 6, 91 − 3	87
25	2474	123 + 6, 130 − 4	128
31	2480	130 + 0	130
Sept. 7	2487	Not seen.	
23	2503	,,	
24	2504	130 + 7, 134 + 0	136
Oct. 7	2517	Not seen < 123 + 4	
20	2530	,, < 123 + 3	
Nov. 12	2553	,, < 123 + 3	
1866			
Jan. 18	2620	Not seen.	
Feb. 14	2647	< 130 + 3	
20	2653	Invisible < 130 + 3	
Mar. 13	2674	Not seen < 130	
Apr. 13	2705	,, < 132	
22	2714	Glimpsed with difficulty, 132	
25	2717	< 128	
May 7	2729	130 + 3	133
15	2737	Not seen < 130 + 2	
17	2739	,, < 130 + 3	
20	2742	130 + 4	134
22	2744	Occasionally glimpsed.	
June 8	2761	Glimpsed.	
25	2778	Not seen < 107	
July 11	2794	,, < 123 + 2	
19	2802	,, < 130	
22	2805	< 130 + 3	
30	2813	Not seen < 130	
Aug. 20	2834	Not seen < 115	

TABLE X.—*Baxendell's Observations of R Coronæ*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1866			
Aug. 31	2845	< 132	
Sept. 15	2860	< 130	
Oct. 6	2881	Not visible < 130	
10	2885	,, < 128	
28	2903	Not seen < 130	
Nov. 6	2912	< 123 + 4	
1867			
Feb. 1	2999	Not visible < 130	
Mar. 26	3052	Not seen < 130	
Apr. 11	3068	< 130	
21	3078	Not seen < 130	
May 14	3101	123 + 0	123
21	3108	123 + 2	125
June 5	3123	123 + 0	123
29	3147	114 + 3	117
July 24	3172	91 + 5, 106 − 4, 97 − 2	94
27	3175	91 + 2, 106 − 7, 97 − 3	95
Aug. 1	3180	91 − 1	90
6	3185	91 − 3	88
21	3200	80 + 3, 91 − 7	84
23	3202	80 + 3	83
Sept. 2	3212	80 − 1, 91 − 9	80
5	3215	80 − 2	78
10	3220	80 − 3, 91 − 9	80
16	3226	80 − 2	78
23	3233	80 − 3	77
Oct. 3	3243	80 − 2	78
22	3262	72 + 4, 80 − 3	76
Nov. 19	3290	72 + 6, 80 − 2, 91 − 11	79
1868			
Mar. 14	3406	80 + 9, 91 − 3	88
Apr. 2	3425	< 106	
9	3432	114 + 8, 123 − 4	120
11	3434	114 + 9, 123 − 3	122
23	3446	123 + 7	130
1868			
May 11	3464	130 + 4	134
30	3483	Not seen < 125	
June 7	3491	,, < 123 + 4	
12	3496	,, < 132	
15	3499	,, < 132	
18	3502	130 + 5	135
July 23	3537	Not seen < 130	
25	3539	Invisible < 132	
27	3541	Glimpsed.	
Sept. 4	3580	Not seen < 125	
Oct. 17	3623	,, < 132	
Nov. 8	3645	< 13	
1869			
Mar. 6	3763	Not visible < 103	
Apr. 12	3800	,, < 133	
30	3818	Occasionally glimpsed.	
June 8	3857	Not seen < 114 + 4	
1877			
Sept. 9	6872	56 + 0	56
15	6878	56 + 1	57
16	6879	56 + 1	57
21	6884	56 + 2	58
Oct. 8	6901	56 + 8	64
13	6906	Invisible [with opera-glass] < 70	
1878			
Aug. 8	7205	97 + 0	97
10	7207	97 − 1	96
21	7218	97 − 5	92
25	7222	97 − 5, 91 − 1	91
Sept. 9	7237	80 + 3, 97 − 8, 91 − 2	87
24	7252	80 + 3	83
Oct. 13	7271	72 + 6, 80 − 1	78
26	7284	72 + 3, 80 − 1	77
Nov. 14	7303	72 − 2	70
1879			
Feb. 28	7409	72 − 4	68

TABLE X.—*Baxendell's Observations of R Coronæ*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1879			
Mar. 5	7414	72 – 6	66
29	7438	72 – 5	67
Aug. 11	7573	72 + 1	73
30	7592	72 – 4	68
Sept. 23	7616	72 – 6	66
Oct. 15	7638	72 – 5	67
Nov. 2	7656	72 – 4	68
1880			
Feb. 14	7760	Glimpsed 135	
Mar. 18	7793	80 + 2, 91 – 8	82
21	7796	80 + 5, 91 – 4	86
Apr. 1	7807	72 + 2	74
11	7817	72 – 1	71
19	7825	72 – 2	70
24	7830	72 – 1	71
May 3	7839	72 + 1	73
16	7852	72 + 4	76
24	7860	72 + 3	75
June 4	7871	72 – 1	71
17	7884	72 – 3	69
July 4	7901	72 + 1	73
9	7906	72 + 1	73
31	7928	72 – 1	71
Aug. 6	7934	72 – 1	71
28	7956	72 – 4	68
30	7958	72 – 6	66
Sept. 5	7964	72 – 3	69
8	7967	72 – 5	67
27	7986	72 – 6	66
Oct. 10	7999	72 – 5	67
29	8018	72 – 6	66
1881			
Apr. 6	8177	72 – 9	63
May 7	8208	72 – 7	65
10	8211	72 – 9	63
20	8221	72 – 9	63
29	8230	72 – 9	63
June 12	8244	72 – 8	64
18	8250	72 – 10	62
July 8	8270	72 – 10	62
14	8276	72 – 9	63
Aug. 1	8294	72 – 10	62
Oct. 16	8370	72 – 10	62
29	8383	72 – 11	61
1882			
Mar. 12	8517	72 – 7	65
Apr. 6	8542	72 – 10	62
May 14	8580	72 – 7	65
Aug. 19	8677	72 – 9	63
Sept. 9	8698	72 – 10	62
1883			
Mar. 5	8875	72 – 5	67
Apr. 3	8904	56 + 2	58
8	8909	72 – 9	63
May 4	8935	56 + 7	63
1884			
Mar. 16	9252	72 – 8	64
Apr. 7	9274	72 – 11	61
28	9295	72 – 9	63
July 2	9360	72 – 7	65
23	9381	72 – 9	63
Aug. 11	9400	72 – 5	67
Sept. 20	9440	72 – 8	64
Oct. 1	9451	72 – 11	61
1885			
Mar. 11	9612	72 – 11	61
27	9628	72 – 9	63
Apr. 18	9650	72 – 8	64
June 1	9694	72 – 9	63
9	9702	72 – 10	62
July 4	9727	72 – 7	65
26	9749	72 – 7	65
Aug. 14	9768	72 – 9	63
1886			
Mar. 8	9974	72 – 8	64
1887			
	241+		
Feb. 18	0321	72 – 7	65

The only regular feature about the variations of this star seems to be that its decrease from maximum is usually sudden and rapid, while its increase from minimum is much more gradual. But it sometimes remains at or near either maximum or minimum for over a year, while at other times it does so for a few days only. On the latter occasions it does not reach so high a maximum or so low a minimum as usual. The following Table shows the fluctuations during the course of these observations, down to 1880 August. After that, all the observations give the star at or near maximum, but the gaps are quite long enough to allow of several minima having occurred.

At or near Max.	Rapid Decrease.	At or near Min.	Increase.
— — to 1851 Aug.	1851 Sept.	1851 Sept. to 1852 Mar.	1852 Apr. to Sept.
1852 Sept. to — —	1855 (?)	— — to 1855 Apr.	1855 May to Oct.
1855 Oct. to 1858 Mar.	1858 Apr.	1858, end of Apr.	1858 Apr. to Aug.
1858 Aug. to 1859 Mar.	1859 Mar. and Apr.	1859 May and June.	1859 June to 1860 Feb.
1860 Feb. to 1861 June.	1861 July.	1861 July and Aug.	1861 Sept. to 1862 Mar.
1862 Mar. to 1863 Oct.	1863 Nov. and Dec.	1864 Jan. to 1865 Feb.	1865 Mar. to July.
1865 July.	1865 Aug.	1865 Aug. to 1867 June.	1867 June to — —
?	(Before 1868 Apr.)	1868 Apr. or earlier to 1869 June or later.	
— — to 1887 Oct. or later.	?	?	— — to 1879 Mar.
1879 Mar. to — —	?	— — to 1880 Feb.	1880 Mar. to Aug.

Observations earlier than these are all at or near maximum, but there are several gaps in which minima may have occurred. One may also have occurred late in 1853.

No. 5, *S Coronæ* (5504 *Chandler*, 151731 *Harvard*).

$15^h\ 17^m\ 19^s \quad +31^\circ\ 43'{\cdot}6\ (1900{\cdot}0).$
$15\ \ 15\ \ 29 \quad +31\ \ 53'{\cdot}5\ (1855{\cdot}0).$

Identification of Comparison Stars. — In this case Baxendell changed his designations of the three faintest comparison stars after the end of 1865, but their identification presents no difficulty.

TABLE XI.

Baxendell.		A. S. V.		Harv. Mag.	Adopted Mag.
1864–5.	From 1866.	No.	Grad.		
a	*a*	2	3	6·96	70
b	*b*	4	17	7·92	80
c	*c*	5	26	8·50	84
d	*d*	6	30	8·73	88
...	*e*	9	42	9·40	94
...	*f*	10	62	10·40	104
x	*g*	14	89	11·65	116
y	*h*	18	107	12·26	123
z	*k*	20	116	12·54	125

The Harvard Mag. is from *H.A.*, xxxvii. p. 194, with interpolations.

TABLE XII.

Baxendell's Differences between the Comparison Stars. Unit 0·1 Magnitude throughout. Subscript figures give number of Observations.

Years.	80–70.	84–80.	88–84.	94–88.	104–94.	123–116.	125–123.
1864–5	13_{13}	7_1	4_1	...	...	8_4	6_3
1866	11_4	4_{10}	...	...	19_1	8_5	3_3
1867–9	12_5	5_2	5_1	2_1	12_2	8_3	2_1
1878	10_7	4_5	6_4	...	...	...	...
1879–80	11_9	2_1	4_1	5_4	12_4	7_3	...
1881–2	...	2_1	7_3	6_3	8_2	7_2	...
1883–4	14_5	6_3	5_3	4_4	16_4	...	...
1885–7	16_5	5_2	...	7_2	...	12_2	1_2
Means	12	4	6	5	13	8	3
No. Obs.	48	25	13	14	13	19	9
Reduced Means.	10	4	5	4	10	7	3

The sum of Baxendell's means is 51, and that of the differences at the head of the columns is 43. The "reduced means" are obtained by the application of the formula $t = 0{\cdot}915s - 0{\cdot}0115s^2$ to each of Baxendell's comparisons individually. Its application to each mean would have little value. The sum of these reduced means is 43, which agrees exactly with that of the differences at the head of the columns, which are nearly in accordance with the Harvard magnitudes.

TABLE XIII.

Baxendell's Observations of S Coronæ.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.	Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1864				1864			
May 26	2018	116 + 7	123	Aug. 23	2107	70 + 0	70
29	2021	116 + 8	124	26	2110	70 + 0	70
July 8	2061	116 – 18, 80 – 5	86	Sept. 5	2120	70 + 1	71
11	2064	70 + 4, 80 – 5	74	11	2126	70 + 0	70
13	2066	70 + 3, 80 – 6	74	22	2137	70 + 1	71
16	2069	70 + 3, 80 – 6	74	24	2139	70 + 1	71
19	2072	70 + 2, 80 – 10	70	Oct. 3	2148	70 + 2	72
26	2079	70 – 1, 80 – 13	68	31	2176	70 + 9, 80 + 0	80
29	2082	70 – 1	69	1865			
Aug. 21	2105	70 – 2	68	Apr. 2	2329	123 + 6	129

TABLE XIII.—*Baxendell's Observations of S Coronæ*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1865			
Apr. 15	2342	123 + 4, 125 − 2	125
27	2354	123 + 3	126
29	2356	123 + 3	126
May 15	2372	123 + 0	123
18	2375	123 + 0	123
20	2377	123 − 1	122
24	2381	123 − 3	120
June 8	2396	116 + 1	117
11	2399	116 + 1, 123 − 7	116
13	2401	116 + 0	116
17	2405	116 − 1	115
20	2408	116 − 4	112
23	2411	88 + 6, 116 − 7	96
28	2416	80 + 5, 88 − 4, 84 − 1	84
July 1	2419	70 + 9, 80 − 3	78
2	2420	70 + 7, 80 − 4	76
11	2429	70 − 1	69
13	2431	70 − 1	69
14	2432	70 − 1	69
19	2437	70 − 2	68
21	2439	70 − 1	69
Aug. 3	2452	70 + 1	71
4	2453	70 + 0	70
8	2457	70 + 0	70
12	2461	70 + 1	71
15	2464	70 + 1	71
25	2474	70 + 2	72
31	2480	70 + 2	72
Sept. 7	2487	70 + 3	73
23	2503	70 + 6, 80 − 6	75
24	2504	70 + 5	75
28	2508	70 + 7, 80 − 3	77
Oct. 7	2517	70 + 9, 80 − 3	78
20	2530	70 + 10, 80 − 2	79
Nov. 12	2553	70 + 14, 116 − 18, 88 + 0	90

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1866			
Feb. 14	2647	116 + 7, 125 − 3, 123 − 1	122
20	2653	116 + 6, 125 − 3, 123 − 1	122
Mar. 13	2674	125 + 2	127
Apr. 13	2705	125 + 4	129
22	2714	125 + 2	127
25	2717	125 + 2	127
May 7	2729	125 + 0	125
15	2737	116 + 7, 123 − 2	122
17	2739	123 + 2, 125 − 3	124
18	2740	123 + 0	123
22	2744	123 − 1	122
26	2748	116 + 6, 123 + 0	122
June 8	2761	116 + 4, 123 − 2	120
25	2778	104 + 0	104
26	2779	104 − 2	102
July 11	2794	84 + 1	85
16	2799	70 + 7, 80 + 2, 84 − 2	80
19	2802	70 + 9, 80 − 1, 84 − 3	80
21	2804	70 + 9, 80 − 2, 84 − 3	79
30	2813	70 + 8, 80 + 0, 84 − 3	80
Aug. 20	2834	80 + 3, 84 − 2	82
27	2841	80 + 2, 84 − 1	82
31	2845	80 + 2, 84 − 1	82
Sept. 15	2860	84 + 1	85
17	2862	80 + 3, 84 − 1	83
Oct. 6	2881	84 + 1	85
14	2889	84 + 3	87
Nov. 6	2912	84 + 9, 94 + 5, 104 − 10	95
1867			
Feb. 1	2999	116 + 3, 123 − 2	120
Apr. 11	3068	123 + 0	123

TABLE XIII.—*Baxendell's Observations of S Coronæ*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1867			
Apr. 21	3078	123 + 2, 125 + 0	125
May 11	3098	116 + 0	116
14	3101	116 + 2	118
21	3108	116 + 3, 123 − 6	118
June 5	3123	104 + 0	104
29	3147	70 + 10, 84 − 2	81
July 24	3172	70 + 0	70
27	3175	70 + 0	70
Aug. 1	3180	70 + 1	71
6	3185	70 + 0	70
21	3200	70 + 0	70
Sept. 2	3212	70 + 3	73
5	3215	70 + 3	73
10	3220	70 + 3, 80 − 6	74
16	3226	70 + 4, 80 − 5	74
Oct. 3	3243	80 + 3, 84 − 3,	82
22	3262	84 + 9, ?96 − 3, 94 + 0	93 or 94
Nov. 19	3290	104 − 6	98
1868			
Apr. 9	3432	123 − 2	121
May 30	3483	104 − 2	102
June 5	3489	104 − 5	99
12	3496	94 + 2, 104 − 9	96
15	3499	94 − 1	93
19	3503	84 + 3, 88 − 2, 94 − 3	88
25	3509	80 + 2, 84 − 2	82
28	3512	70 + 10, 80 − 1	80
July 11	3525	70 + 2, 80 − 8	72
13	3527	70 + 1	71
21	3535	70 − 2	68
23	3537	70 − 2	68
25	3539	70 − 1	69
29	3543	70 + 0	70
Aug. 3	3548	70 + 1	71
Sept. 4	3580	70 + 6, 80 − 3	76
Nov. 8	3645	94 + 2 104 − 7	96
1869			
Apr. 12	3800	123 + 0	123
30	3818	116 + 3, 123 − 3	120
June 8	3857	104 − 3	101
1878			
Apr. 8	7083	116 + 2	118
May 4	7109	70 + 5, 80 − 3	76
5	7110	70 + 5, 80 − 3	76
11	7116	70 + 2, 80 − 6	73
21	7126	70 + 0	70
30	7135	70 + 0	70
June 1	7137	70 + 1	71
5	7141	70 + 1	71
10	7146	70 + 1	71
23	7159	70 + 3, 84 − 9	74
25	7161	70 + 2, 84 − 9	74
28	7164	70 + 3, 84 − 8	74
29	7165	70 + 3	73
July 7	7173	70 + 4, 84 − 7	76
8	7174	70 + 4	74
18	7184	70 + 6, 84 − 7	76
19	7185	70 + 7, 84 − 7, 80 − 2	77
28	7194	70 + 10, 84 − 2	81
31	7197	70 + 11, 84 − 3	81
Aug. 6	7203	70 + 11, 80 + 2, 84 − 1	82
8	7205	84 − 2	82
10	7207	84 − 1	83
18	7215	84 + 2	86
25	7222	84 + 3	87
Sept. 3	7231	84 + 7, 88 + 2, 104 − 9	92
9	7237	84 + 6, 88 + 2	90
11	7239	84 + 8, 88 + 3	92
13	7241	84 + 8, 88 + 2	91
24	7252	88 + 6	94
Oct. 13	7271	104 − 4	100
26	7284	104 + 1	105

TABLE XIII.—*Baxendell's Observations of S Coronæ*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1879			
Feb. 28	7409	(About 13 mag.)	
Mar. 5	7414	Not seen < 123	
29	7438	116 + 2, 123 − 1	120
Apr. 6	7446	104 + 8, 116 − 3	112
Aug. 11	7573	84 + 1, 88 − 2	86
23	7585	88 + 4, 94 − 3, 104 − 10	92
26	7588	94 + 0, 104 − 8	95
30	7592	88 + 3, 94 − 1	92
Sept. 7	7600	94 + 7, 104 − 5	100
23	7616	104 − 1	103
25	7618	104 − 2	102
Oct. 15	7638	116 − 3	113
Nov. 2	7656	116 + 1, 123 − 7	116
1880			
Feb. 14	7760	125 + 5	130
Mar. 18	7793	116 + 4, 123 − 2	120
Apr. 1	7807	104 + 8, 116 − 3	112
11	7817	104 + 0	104
16	7822	104 − 7	97
19	7825	88 + 0, 94 − 3	90
24	7830	84 − 1	83
May 3	7839	70 + 5, 80 − 1	77
13	7849	70 + 5, 80 − 5	75
16	7852	70 + 3, 80 − 4	74
24	7860	70 + 4, 80 − 6	74
June 4	7871	70 + 6, 80 − 6	75
8	7875	70 + 4, 80 − 3	76
17	7884	70 + 6, 80 − 5	76
July 4	7901	70 + 7, 80 − 1	78
9	7906	70 + 9, 80 − 2	78
20	7917	80 + 2	82
31	7928	80 + 1	81
Aug. 6	7934	80 + 1	81
9	7937	80 + 3, 84 + 1	84
14	7942	84 + 0, 94 − 5	86
28	7956	88 + 2, 94 − 4	90
Sept. 5	7964	94 − 1	93
Sept. 16	7975	94 + 4, 104 − 7	98
27	7986	104 − 3	101
Oct. 29	8018	104 + 5, 116 − 4	110
1881			
Mar. 31	8171	88 − 3	85
Apr. 6	8177	84 + 2, 88 − 2	86
May 7	8208	70 − 2	68
10	8211	70 − 3	67
20	8221	70 − 3	67
29	8230	70 − 3	67
June 12	8244	70 − 4	66
18	8250	70 − 1	69
27	8259	70 − 1	69
July 8	8270	70 + 1	71
11	8273	70 + 3	73
14	8276	70 + 1	71
17	8279	70 + 3	73
20	8282	70 + 3	73
27	8289	70 + 3	73
Aug. 1	8294	70 + 5	75
6	8299	70 + 8	78
24	8317	84 + 2, 88 − 2, 94 − 6	87
Sept. 14	8338	88 + 8, 104 − 7, 94 + 3	97
Oct. 16	8370	104 + 3	107
29	8383	116 − 2	114
1882			
Mar. 12	8517	116 + 4, 123 − 1	121
16	8521	116 + 6, 123 − 1	122
Apr. 6	8542	84 + 9, 104 − 11, 88 + 0	91
10	8546	88 − 3, 84 + 2, 104 − 10	88
20	8556	80 − 2, 84 − 3	80
May 4	8570	70 + 10, 84 − 7	78
14	8580	70 + 4, 84 − 6	76
20	8586	70 + 7, 84 − 3	79

TABLE XIII.—*Baxendell's Observations of S Coronæ*—continued.

Date.	J.D. 240+.	Corrected Comparisons.	Deduced Mag.
1882			
May 30	8596	70 + 7, 84 − 5	78
July 24	8651	84 + 3, 94 − 3	89
Aug. 19	8677	94 + 0, 104 − 5, 88 + 4	95
Sept. 9	8698	104 − 5	99
1883			
Mar. 4	8874	116 + 0	116
24	8894	88 + 2, 94 − 3	90
Apr. 8	8909	84 − 4	80
1884			
Feb. 25	9232	116 + 2	118
Mar. 16	9252	104 − 4	100
21	9257	88 − 1, 94 − 3	89
25	9261	84 + 1, 94 − 8, 88 − 6	84
Apr. 7	9274	70 + 6, 80 − 6	75
9	9276	70 + 4, 80 − 7	74
10	9277	70 + 4, 80 − 7	74
12	9279	70 + 3	73
28	9295	70 − 2	68
May 4	9301	70 − 1	69
18	9315	70 + 4	74
22	9319	70 + 4	74
24	9321	70 + 2	72
27	9324	70 + 3	73
June 4	9332	70 + 1	71
11	9339	70 + 4	74
20	9348	70 + 8, 80 − 5	76
July 2	9360	70 + 9, 80 − 2	78
23	9381	80 + 3, 84 − 1	83
Aug. 11	9400	94 + 2, 104 − 11	94
19	9408	94 − 1, 104 − 8	94
Sept. 4	9424	94 + 7, 104 − 10	98
20	9440	104 − 2	102
Oct. 1	9451	104 − 2	102
Nov. 19	9500	123 − 2	121
1885			
Mar. 11	9612	116 + 0	116
27	9628	94 + 0	94
Apr. 1	9633	88 + 1, 94 − 3	90
8	9640	84 + 0	84
17	9649	80 − 2	78
18	9650	80 − 3	77
23	9655	70 + 9, 80 − 4	78
29	9661	70 + 8, 80 − 6	76
May 7	9669	70 + 9, 88 − 4	78
10	9672	78 + 7, 80 − 6	76
16	9678	70 + 9, 80 − 3	78
25	9687	80 − 3	77
June 1	9694	80 + 0	80
9	9702	80 + 3, 84 − 2	82
12	9705	80 + 0, 84 − 3	80
July 4	9727	84 + 3	87
26	9749	88 + 6, 94 − 2	93
Aug. 14	9768	104 − 3	101
Sept. 27	9812	116 − 3	113
1886			
Feb. 14	9952	125 + 1	126
26	9964	116 + 1	117
Mar. 8	9974	116 − 4	112
29	9995	84 + 4	88
Apr. 6	241+ 0003	84 − 3	81
26	0023	70 + 3, 84 − 8	74
May 21	0048	70 + 4, 84 − 3	78
29	0056	70 + 6, 84 − 6	77
June 30	0088	84 − 1	83
Sept. 17	0167	104 − 2	102
Nov. 2	0213	116 + 3, 123 − 5	118
17	0228	116 + 5, 123 − 5	120
1887			
Feb. 18	0321	104 + 2	106

The results of these observations are summarised in Table XIII. as in the preceding cases. Taking the simple means, as given at

the foot of the columns, we find the following corrections to epoch at the three mean dates:—

For	I.+(4)	mean E = 6	correction to epoch =	$-2^d\cdot0$
,,	XV.+(4)	,, = 20	,,	,,	$+2\cdot2$
,,	XX.+(4)	,, = 25	,,	,,	$+3\cdot4$.

If we weight the means according to the number of observations, the first correction becomes $-2\cdot6$, and the second $+3\cdot1$, while the third is not changed. But as the differences seem to be mainly due to some abnormally large residuals in a few individual periods, it seems better to keep to the simple means. In any case, these differences are but small.

TABLE XIV.

S Coronæ. Assumed Period, 360 *Days.*

Group.	Mag. from Curve. Julian Date 240+.	123. 15.	108. 45.	78. 75.	73. 105.	75. 135.	82. 165.	93. 195.	99. 225.	108. 255.	120. 285.	124. 315.	125. 345.
I.	2000	$+1_2$	...	-7_7	-4_3	-4_5	-6_1	...	...	...	...	$+4_1$	$+1_3$
II.	2360	-1_4	0_8	-11_6	-3_6	-1_5	-3_2	-2_1	...	...	$+1_2$	$+3_1$	$+3_3$
III.	2720	$+1_6$	$+12_3$	$+4_4$	$+8_2$	$+8_4$	$+4_2$	$+3_1$	...	...	$+2_1$	...	-1_2
IV.	3080	-4_3	-6_1	-1_1	-4_4	-2_5	$+1_1$	$+5_1$	$+2_1$	...	...	...	-4_1
V.	3440	...	-1_4	-1_5	-4_5	$+1_1$	...	$+1_1$	...	...	...	...	...
VI.	3800	-2_2	$+10_1$	...	...	...	...	...	...	...	...	...	...
XV.	7040	...	$+8_1$	-5_4	-2_5	0_7	$+1_6$	-2_5	-2_2	$+1_1$	...	...	...
XVI.	7400	...	$+6_2$	...	...	...	$+1_1$	$+3_4$	$+3_3$	$+8_1$	...	...	...
XVII.	7760	$+5_1$	$+8_3$	$+4_5$	$+1_4$	$+2_3$	-1_4	-2_3	$+1_2$	0_1	...	...	...
XVIII.	8120	...	-11_2	-7_1	-7_3	-6_3	-8_8	-6_1	-1_1	$+2_2$	...	..	...
XIX.	8480	...	$+8_2$	$+4_3$	$+4_4$	..	$+5_1$	$+2_1$	$+1_1$	...	...	...	...
XX.	8840	...	-3_2	-1_1	...	...	...	...	...	...	...	...	...
XXI.	9200	...	0_3	-4_5	-2_4	-1_5	-2_1	-2_3	-2_1	-3_2	...	-2_1	...
XXII.	9560	...	$+19_1$	$+8_4$	$+3_6$	$+5_4$	$+5_1$	$+4_2$	...	$+6_1$	...	...	...
XXIII.	9920	...	$+8_3$	$+7_2$	$+1_1$	$+3_2$	0_1	...	...	-3_1	-3_1	-4_1	...
XXIV.	241+ 0280	...	-6_1	...				...	...			...	...
I.+(4)		−1	+1	−3	−1	0	−1	+2	+2	...	+1	+4	0
XV.+(4)		+5	+4	−1	−1	−1	0	−1	0	+3	(0)	(0)	(0)
XX.+(4)		(0)	+4	+2	+1	+2	+1	+1	−2	0	−3	−3	(0)
	$p-f$	+17	+44	+35	+4	−9	−18	−19	−15	−20	−16	−5	+1

The epoch of maximum as estimated from the smooth curve used in forming the residuals (indicated by the mean magnitudes at the head of the columns) falls about 112 days after the adopted

commencement of first portion, or 97 days after its mean date, *i.e.* at the following dates :—

Curve.	Correction.	240+.	E.	Chandler.	Bax. – Ch.
d d	d			d d	d
2402735 + 97	– 2·0	= 2830·0	6	2814·2 + 5·0	+ 10·8
7775 + 97	+ 2·2	= 7874·2	20	7871·0 – 3·6	+ 6·8
9575 + 97	+ 3·4	= 9675·4	25	9677·0 – 8·0	+ 6·4

The Rousdon observations (*Mem. R.A.S.*, lv. p. 83) are not expressed in quite this form, but it is easy so to express them, as follows :—

Rousdon.	241+.	E	Chandler.	R. – Ch.
2411072 + 45 – 11·9	= 1105·1	29	1121·8 – 5·7	– 11·0
2414312 + 45 + 20·2	= 4377·2	38	4372·6 + 7·1	– 2·5

Chandler's formula is $2400647 + 361{\cdot}2E + 8\sin(12^\circ E + 327^\circ)$ and the periodic term is shown separately. The mean difference between Baxendell and Rousdon is due to the essential difficulty of assigning the maximum, and need not be too seriously regarded. It affords a good illustration of the difficulty of combining the observations of different observers when the epoch of maximum is taken as the variable. But the *progression* of the differences in the two sets indicates that the coefficient 8 is too small, since an increase would make both sets more accordant. In *Monthly Notices*, lxviii. p. 552, a relation is found between this coefficient C and the period P, which for $P = 360^d$ would give $C = 30$ instead of 8 ; and in the same paper the value of A appropriate to $P = 360$ is found to be $9^\circ{\cdot}1$ instead of 12°. The value of A is shown to be better determined. As an experiment, let us adopt the theoretical value $9^\circ{\cdot}1$ for A and find the value of C on the assumption that both the Baxendell and Rousdon maxima require a constant epoch correction. We thus have

E.	$\phi = 9^\circ{\cdot}1$ E.	Equations.
	°	d
6	55	+ ·82 S + ·57 T = + 15·8 + B
20	182	– ·03 S – 1·00 T = + 3·2 + B
25	228	– ·74 S – 0·67 T = – 1·6 + B
29	264	– ·99 S – 0·10 T = – 16·7 + R
38	346	– ·24 S + 0·97 T = + 4·6 + R

the equations above shown to find S and T, where $S = C\cos\omega$ and $T = C\sin\omega$ in the expression

$$C\sin(9^\circ{\cdot}1\ E + \omega).$$

Subtracting third from first to eliminate B, and also fourth from fifth, we find

$$S = -10{\cdot}6, \quad T = +27{\cdot}4,$$

whence $C = 29{\cdot}4$, close to the theoretical value indicated above. Taking these values of S and T, we find

$$B = -9, \ -30, \ -9, \quad R = +24, \ +24.$$

The middle group of B is, however, quite discordant, and the difference B – R is too large. By the help of Knott's observations (*Mem. R.A.S.*, lii. p. xvii) and some later Rousdon observations (still in MS.), the following formula is suggested:—

$$2400640 + 361{\cdot}50\mathrm{E} + 12 \sin (9^{\circ}{\cdot}5\mathrm{E} + 14^{\circ}).$$

If we analyse the results harmonically, we get for the adopted curve

$$2{\cdot}63 \cos (\theta - 303^{\circ}) + 0{\cdot}65 \cos 2(\theta - 352^{\circ}) + 0{\cdot}26 \cos 3(\theta - 17^{\circ}),$$

or referring this to the maximum ($\theta = 97^{\circ}$) as zero point, we get

$$2{\cdot}63 \cos (\theta - 206^{\circ}) + 0{\cdot}65 \cos 2(\theta - 255^{\circ}) + 0{\cdot}26 \cos 3(\theta - 280^{\circ}).$$

The Rousdon curve for S Coronæ is given in *Mem. R.A.S.*, lv. p. xcix, Table LIV.—

$$-1{\cdot}15 \sin\theta - 2{\cdot}51 \cos\theta + 0{\cdot}59 \sin 2\theta - 0{\cdot}46 \cos 2\theta + 0{\cdot}13 \sin 3\theta + 0{\cdot}21 \cos 3\theta,$$

but these coefficients have been divided by the range (5·0) and multiplied by 6. To compare with Baxendell we therefore multiply by 5/6, and the expression may then be written

$$2{\cdot}32 \cos (\theta - 205^{\circ}) + 0{\cdot}62 \cos 2(\theta - 244^{\circ}) + 0{\cdot}21 \cos 3(\theta - 251^{\circ}).$$

The following report was received too late for insertion in the Annual Report of the Council.

Melbourne Observatory. (*Director, Mr. P. Baracchi.*)

Meridian observations made with the 8″ Transit Circle.

Objects observed.	Number of Observations.	
	R.A.	N.P.D.
Clock Stars	573	262
Azimuth Stars	318	198
List Stars	1329	1327
Totals	2220	1787

The list stars in the above table form part of a supplementary working list of 2449 reference stars required for the determination of plate-constants of the Melbourne astro-photographic zones. About one half of their total number still remains to be observed.

Astrographic Work.—Since the death of Mr. Johns no regular observer has been appointed to replace him, in consequence of which the astrograph has been employed only on a few occasions.

The total number of plates was sixteen, of which four were rejected as faulty, the others being distributed as follows, viz.:—

Two catalogue plates to replace others, of the same regions, which had been rejected.
Three plates of Oxford chart regions.
One plate of the South Pole region.
Two plates for adjustment.
Four plates of "Kapteyn selected areas," with exposures of 6m. and 20m. for both the polar region and the area in declination $-75°$.

The measurement of the plates in the Melbourne zones has been completed, and the reduction of the measures and preparation for printing have been proceeded with.

During the year the investigation of the Réseau Melbourne No. 6 was completed, and the errors were determined also of Réseaux Melbourne No. 11, Gautier No. 23, and Gautier No. 27.

The maximum corrections (irrespective of sign) are:—

Melbourne	No. 23,	0·013 mm. (Previously investigated.)
,,	No. 6,	·008
,,	No. 11,	·010
Gautier	No. 33,	·002
,,	No. 27,	·002

The corrections to Melbourne No. 11 and Melbourne No. 23 are largely due to want of perfect rectangularity in the two systems of lines, and those of Réseau Melbourne No. 6 to errors in spacing of the lines. In all cases the perfect straightness of the lines is very closely approached.

8-inch Equatorial Telescope.—With this telescope observations with the square-bar micrometer of Comet Zinner (1913 *a*) were obtained on seven nights.

The comet announced by Lowe in January was searched for on three nights unsuccessfully.

Some few other occasional observations were made with this and the smaller equatorial.

Wireless time-signals were sent from the Melbourne Observatory to Adelie Land through Macquarie Island, for the determination of longitude of Dr. Mawson's Station in Antarctica, each night from February 4 to February 14; also on February 25 and (from the Wireless Station in the Domain, by hand signals with chronometer) on December 1, 2, 3, but in no case were time-signals received at Melbourne from Adelie Land.

Since September 10, time-signals have been sent daily at 1 p.m. to the Wireless Station in the Domain (a few hundred yards from the observatory), and from thence retransmitted automatically for the convenience of ships at sea.

The installation for transmitting daily time-signals by wireless in accordance with the international programme is being completed,

and this observatory is now ready to take part in this service on behalf of the Commonwealth of Australia.

In 1911 September, the Commonwealth Government having erected at Mount Stromlo, within the Federal Territory, a structure of concrete with an 18-feet dome suitable for installing a 9-inch telescope by Grubb (presented by the late Mr. James Oddie, F.R.A.S., of Ballarat), requested the Director to erect the instrument, and to arrange a programme of observations to be made at this place in the course of at least one year, for the purpose of determining whether the site was sufficiently satisfactory to justify the establishment of a modern first-class astronomical observatory, including a special department for the study of the sun.

The programme included meteorological and astronomical observations made with the naked eye and with the telescope, also photographs of nebulæ and stellar regions, and delineation of solar prominences. In order to carry out this work, a 6-inch Dallmeyer photographic portrait lens, mounted in a metallic star camera, was attached to the telescope, and an Evershed solar prominence spectroscope was also fitted for the purpose.

Monthly visits were paid to the locality by the Director and chief assistant, Dr. Baldwin, from 1911 September to 1913 May, each visit lasting about a week.

The results were embodied in a report which was presented to the Commonwealth authorities last June, in which it is shown that the site was found to be sufficiently satisfactory for the proposed observatory.

No further steps, however, have been taken by the authorities, and the present Mount Stromlo Observatory, having accomplished its task, has since remained inoperative.